Macroeconomic Disagreement in Treasury Yields

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Abstract

I estimate a term structure model of Treasury yields in which traders' information about macroeconomic conditions is dispersed. Bond yields and inflation forecasts identify properties of traders' information. I find that prices are moderately informative about economic fundamentals, but more informative about policy and others' beliefs. Nevertheless, beliefs about the macroeconomy are estimated to be quite heterogeneous. Over the sample period, dispersed beliefs directly added an average of 60 basis points to ten year yields, mostly attribute to disagreement about the Federal Reserve's inflation target. Accounting for learning and belief heterogeneity dramatically reduces the magnitude and volatility of risk premia relative to estimates that assume full information.

1 Introduction

Surveys of professional forecasters and market participants suggest there is generally non-trivial disagreement about current and future macroeconomic variables and the evolution of asset prices. How does disagreement about the macroeconomy affect the price of long-term assets? And what can the dynamics of asset prices reveal about beliefs and disagreement? Understanding belief formation in the bond market is central for both formulating realistic models of the term structure and for constructing robust models of the links between real and financial variables central to policy making. Moreover, understanding financial market participants' belief formation can aid the assessment of structural macroeconomic models with dispersed beliefs.

To answer these questions, I estimate an affine term structure model (ATSM) of Treasury yields, where short rates and macroeconomic variables are described by a structural vector autoregression. I relax the usual assumption that agents share a common information set. Instead, I model traders' information as *dispersed*: atomistic bond traders optimally combine noisy, idiosyncratic signals with asset prices to form expectations. Because traders care about other traders' beliefs, they must form expectations of fundamentals, as well as expectations about others' expectations, and about others' expectations of others' expectations, and so on. Given this "forecasting the forecasts of others" problem, the model solution is a fixed point between the evolution of agents' beliefs and the prices that inform those beliefs.

The dispersed information affine term structure model builds on earlier work by Barillas and Nimark (2015). They assume yields are driven by latent factors in the yield curve and identify expectations using interest rate forecasts. By contrast, I explicitly model the relationship between short rates, macroeconomic variables, financial risk, and monetary policy. I generalize the structural VAR of Ireland (2015) to incorporate dispersed information. The central bank is assumed to select a time varying inflation target, and set short rates

according to a Taylor rule. This rule responds to the output gap, deviations in inflation from the target, and financial risk. Shocks to the macroeconomic factors affecting short rates are identified using structural assumptions. Changes in market prices of risk are governed by fluctuations in a single risk variable, consistent with the results of both Cochrane and Piazzesi (2005) and Bauer (2016). Shocks to this variable are correlated with macroeconomic shocks, and the risk variable affects macroeconomic dynamics. Hence, the model allows for an interrelationship between the macroeconomy and financial markets. Traders' information is identified using both yields (which depend on traders' macroeconomic inference problem) and on the distribution of inflation forecasts (which directly depends on signal noise). I estimate the model with data from 1971 to 2007 using full information Bayesian methods.

The estimation allows me to quantify the informativeness of different public and private signals for bond traders, and the relative importance of differences in beliefs about particular macroeconomic variables for yields. To my knowledge, these results are new to the literature. My estimates imply roughly half of what bond traders know about macroeconomic factors (deviations of inflation from the Federal Reserve's target and the output gap) comes from observing asset prices, rather than private signals. Asset prices are more informative about risks related to the central bank's unobserved inflation target, and are the source of nearly everything traders know about the others' beliefs. The short rate, combined with agents' idiosyncratic information, contains nearly all the information agents have about fundamentals and others' beliefs. The importance of the policy rate in expectations formation is consistent with earlier structural and reduced form estimates of a "signaling" channel of policy (for example, Melosi (2017), Tang (2013)).

Using the estimated parameters, I decompose the yield curve. Like in a typical ATSM, I identify a component of bond prices due to expected short rates (the *expectations hypothesis component)* and compensation for risk. Compensation for risk can be separated into two components: a portion unrelated to belief heterogeneity (the *classical risk premium*) and a

wedge in prices due to dispersed information (the *higher order wedge*). This wedge comes from the fact that agents' expectations of others' expectations ("higher order" expectations) differ from average expectations about fundamentals, and hence prices differ from those that would obtain if traders counterfactually held common beliefs.

Once we account for imperfect, dispersed information, classical risk premia are estimated to be quite small and nearly constant, in sharp contrast to their full information counterparts. Instead, the model attributes the vast majority of movement in long-term yields to the expectations hypothesis component. Average short rate expectations adjust more slowly than they would if traders had full information. This is due to the fact that agents' optimal signal extraction problem attributes some changes in fundamentals to noise, and some portion of transitory shocks to persistent changes in the inflation target. The higher order wedge, on average, contributed 60 basis points to ten year yields over the sample period. Because the risks in the model are macroeconomic, this wedge can itself be meaningfully decomposed into macroeconomic components. The majority of time variation in the wedge for long-term debt is attributable to changes in higher-order beliefs about monetary policy, particularly, policymakers' long-run inflation target. The decomposition suggests much of the excess sensitivity of long term yields to short-term macroeconomic news (Gurkaynak et al. (2005)) is attributed by the model to violations of the auxiliary assumption of full information.

My results are particularly related to the branch of the macro-finance literature that relates long maturity bond price movements to changes in the monetary policy framework. Gurkaynak et al. (2005) suggest incorporating learning about a long-run inflation target can help macro-finance models explain the effect of transitory shocks on long-term bonds. This paper extends this idea to the entire term structure. Moreover, I allow for pervasive information frictions about macroeconomic variables. My results complement those of Wright (2011) who links declines in term premia to falling inflation uncertainty due to changes in the conduct of monetary policy. It also complements Doh (2012), who estimates a DSGE

model where agents have a noisy signal of trend inflation. He interprets this noise as imperfect credibility of the inflation target. Like these papers, my paper links changes in long-run policy targets to declines in measured risk premia. Unlike these papers, I incorporate dispersed information, quantify signal informativeness, and jointly estimate the dynamics of macroeconomic variables, beliefs and bond prices.

My finding of small risk premia and persistent short rate expectations stands in contrast to the literature that explains yields under full information rational expectations. My results add to growing evidence that accounting for information frictions tends to make time varying risk premia less important for explaining yields. Critically, the slow adjustment of rate expectations holds even with optimal Bayesian learning where agents have model consistent beliefs and a large number of signals. This stands in contrast to others (e.g. Dewachter and Lyrio (2008)) who assume traders' forecasts are based on a model-inconsistent prior. Moreover, my structural results are consistent with the more agnostic approach of Piazzesi et al. (2013) who construct subjective beliefs without modeling inference. Unlike them, I am able to quantify the information content of different signals.

The findings of this paper should be of interest to researchers working with more dynamic general equilibrium or financial models featuring information frictions. The term structure model makes relatively modest structural and functional form assumptions. Unlike many exogenous information models, I do not restrict agents from learning from prices.[1] The ATSM is consistent with the pricing implications of many equilibrium models: Barillas and Nimark (2015) show the dispersed-information ATSM nests an equilibrium model with wealth-maximizing traders, and a number of authors have also embedded ATSM in DSGE models (for example Jermann (1998), Wu (2001), Doh (2012)). The estimated results point to prices as an important source of information for agents making investment decisions. This

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[1]This is in line with the "market consistent information" assumption advocated by Graham and Wright (2010).

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feature of prices has a long intellectual history but its empirical implications have not been explored as much in the literature on macroeconomic models with dispersed information.

Other related literature   A number of authors have estimated models combining structural macroeconomic models with a no-arbitrage finance model. Most of these models assume agents have full information rational expectations. Ang et al. (2007) estimate Taylor rules in such a setting. Rudebusch and Wu (2008) link yields to a dynamic New Keynesian model. Unlike these papers, I allow for macroeconomic disagreement.

My emphasis on learning from prices means that this paper is closely related to the noisy rational expectations literature, following Grossman and Stiglitz (1980). For example, Hassan and Mertens (2014) embed a Hellwig (1980) noisy rational expectations model in a DSGE model to study the equity premium. My model is less structural than these models to facilitate estimation while retaining a complicated inference problem with many assets and fundamentals.[2]

This paper falls primarily into the recent literature on deviations from full information rational expectations in asset pricing. Much of this literature retains the assumption of common information or assumes agents are not Bayesian learners. Apart from earlier mentioned papers, Piazzesi and Schneider (2007) examine how different assumptions about information affect risk premia in a representative investor setting. Collin-Dufresne et al. (2016) examine how Bayesian learning about parameters related to long-run risks, rare disasters, and model uncertainty can help a general equilibrium model generate realistic risk premia.

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[2]A related, but distinct literature associated with Harrison and Kreps (1978) focuses on agents who "agree to disagree" about fundamentals. Unlike the papers in this literature, I assume any differences in agents' beliefs are driven by differences in observed signals. In this way, the model in this paper is consistent with the "Harsanyi doctrine" (Harsanyi (1968)); agents have full information about the structure of the model and its parameters, and form expectations optimally. Only differences in information gives rise to differences in belief. Moreover, the dispersed information setup avoids the critique of Aumann (1976), who points out that two agents with common priors whose posteriors are common knowledge cannot "agree to disagree." Here, posterior beliefs of particular agents about the state will not be common knowledge, and thus need not be the same despite a common prior. Posterior beliefs about prices will be common knowledge because they are commonly observed.

The asset pricing literature that allows for differences in belief tends to abstract from higher order beliefs, the macroeconomy, or both. Giacoletti et al. (2015) also develop an arbitrage-free term structure model with belief dispersion but explicitly ignore the "forecasting the forecasts of others" problem. Colacito et al. (2016) develop an equity pricing model that includes variance and skewness of professional forecasts, which they treat as exogenous. Makarov and Rytchkov (2012) show the state space of a dynamic asset pricing model with dispersed information can be infinite-dimensional, and that information asymmetries affect the time series properties of returns. Kasa et al. (2014) solve a present value model with higher-order expectations in the frequency domain.

Finally, this paper is related to the literature that explains the beliefs implied by forecast surveys. Examples include Patton and Timmerman (2010), Andrade et al. (2014) and Crump et al. (2016). Like these papers, I use the cross-section of forecasts at different horizons to help identify agents' beliefs. However, I endogenize both forecasts and bond prices. Because asset prices are driven by expectations about the future, they are informative about the beliefs of market participants above and beyond what might be captured in surveys.

Outline of the remainder of the paper The next section 2 presents reduced form evidence of information frictions in financial forecasts. In section 3 I outline the asset pricing side of the model, the macroeconomic VAR. Details of the solution and estimation strategy are in sections 3 and 4. I then discuss the parameter estimates and impulse responses (section 5), the information content of signals (section 6), and the model's interpretation of the sources of yield fluctuations (section 7) before concluding.

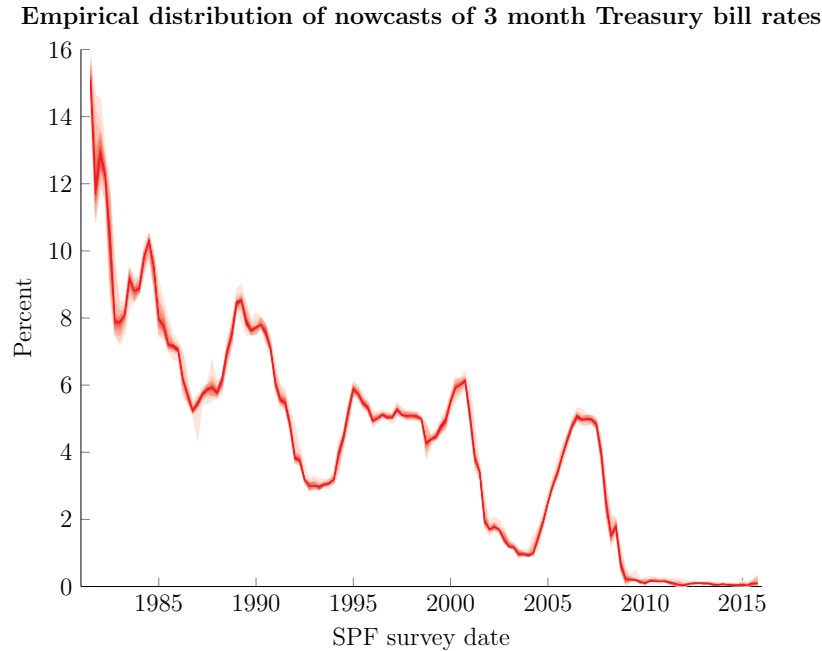**Empirical distribution of nowcasts of 3 month Treasury bill rates**

Figure 1: Distribution of SPF forecasts of *current*-quarter average rate on 3-month Treasury bill.

2   Dispersed Information: Evidence from Forecasts

A number of papers (e.g. Mankiw et al. (2004), Coibion and Gorodnichenko (2012, 2015)) have shown evidence of macroeconomic information frictions using forecast data. With the exception of Coibion and Gorodnichenko (2015), most papers have focused on inflation expectations In this section, I briefly discuss some evidence for the presence of dispersed information about the evolution of Treasury bond prices in particular.

I take data on forecasts from the Survey of Professional Forecasters (SPF). The SPF is a quarterly survey originally conducted by the American Statistical Association and the NBER before being taken over by the Federal Reserve Bank of Philadelphia in 1990. The survey is generally sent out after after the initial release of the National Income and Product Accounts to a panel of forecasters in the financial services industry, non-financial private sector, and academia.

The SPF began to survey its panel about 3-month Treasury bill rates in 1981. The 5th through 95th percentiles of the current-quarter forecasts are shown in figure 1. Despite the fact that the rate for a Treasury bill in the secondary market is observable freely in real time to survey participants, there is still a fair amount of disagreement among forecasters within the current quarter - that is to say, the forecasters surveyed in the SPF disagree about what the average yield of Treasury bills will be over the course of the next two months. The interquartile range of forecasts, even including the zero lower bound period where Treasury bill rates were also effectively zero, is still nearly 20 basis points, with the overall range of forecasts often in the neighborhood of 100-200 basis points. To place these ranges in context, the average yield on 3-month Treasuries was about 436 basis points between 1981-2015. From 2008-2015, the average was 24 basis points. The distribution of one-quarter ahead forecasts, shown in appendix A, are similar.

To more formally test for information frictions, appendix A presents results from applying the empirical strategy of Coibion and Gorodnichenko (2015) to forecasts of bond yields in the SPF.[3] The results in the appendix suggest there is a positive, significant relationship between average forecast errors and average forecast revisions. This is inconsistent with full information rational expectations, or with common, imperfect information. However, it is consistent with a world where agents have dispersed information and thus base forecasts off of different information sets.

Although the SPF forecasts for interest rates are suggestive, there may be two concerns about the regression results. First, it may be that the true beliefs of bond traders are distinct from the beliefs of the survey participants. Hence, in the structural estimation, the beliefs of agents are also identified using both asset prices and forecasts themselves. Second, survey participants may have a different idea in mind of what the relevant bond price to forecast is,

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[3]Coibion and Gorodnichenko (2015) consider Treasury bill forecasts as part of their pooled regressions in a robustness test, but do not explicitly test for information frictions using financial asset forecasts alone.

or may not forecast that variable, which lowers the number of sample responses. Hence, the estimation does not make use of interest rate forecasts, but rather uses inflation forecasts which are regarded as being quite accurate (Faust and Wright (2013)).

3 The dispersed information model and structural VAR

In this section, I outline the model of asset prices and macroeconomic dynamics used to assess the effect of macroeconomic disagreement on prices. The asset pricing intuition and derivation in the next subsection closely follows that of Barillas and Nimark (2015); some additional details are found in appendix B. After outlining the asset pricing model, I discuss the structural VAR.

3.1 The term structure model with heterogeneous information

Intuition: the fundamental asset pricing relationship Index bond traders by $j \in [0,1]$. Denote $E_t^j x_t = E[x_t|\Omega_t^j]$ as the expectation conditional on j's information set at time t (Ω_t^j). Call Ω_t the "full information" information set (i.e., the history of the realizations of all variables up to time t).

Under full information, the basic bond pricing equation is

$$P_t^n = E_t[M_{t+1}P_{t+1}^{n-1}]$$

Standard results in asset pricing theory give that the nominal stochastic discount factor M_{t+1} exists and is positive if the law of one price holds and in the absence of arbitrage (e.g. Cochrane (2005)). If we relax the common information assumption, instead assuming there are a continuum of agents $j \in (0,1)$ with heterogeneous information sets, the pricing relationship for each agent j is:

$$P_t^n = E_t^j \left[M_{t+1}^j P_{t+1}^{n-1} \right] \tag{1}$$

Both information sets (Ω_t^j) and stochastic discount factors (M_{t+1}^j) are j-specific. Centralized trading implies it is common knowledge that all agents face the same prices today and will face the same price tomorrow; because traders are atomistic, they take prices as given. However, allowing information sets and forecasts of future prices to differ across agents, while assuming today's price is common knowledge, implies equation (1) can hold with equality only if stochastic discount factors also differ.

To decide their willingness to pay for a bond, agents must form expectations of future prices. Because future buyers face the same problem, the decision to purchase a bond today depends on a conjecture about others' (future) beliefs - the Townsend (1983) "forecasting the forecasts of others" problem. More specifically, dispersion of information implies asset prices potentially depend on *higher order expectations* - expectations of expectations.[4] Assuming common knowledge of the pricing equation, joint lognormality of prices and stochastic discount factors, and constant conditional variances, one can show (appendix B) the log price of the bond takes the following form:

$$
\begin{aligned}
p_t^n = &\int E_t^j[m_{t+1}^j] dj \\
&+ \int E_t^j \left[\left(\int E_{t+1}^k[m_{t+2}^k] dk + \int E_{t+1}^k[p_{t+2}^{n-2}] dk \right) \right] dj \\
&+ \frac{1}{2} \text{Var}(m_{t+1}^j + p_{t+1}^{n-1}) + \frac{1}{2} \text{Var}(m_{t+2}^k + p_{t+2}^{n-2})
\end{aligned}
$$

The price of a bond in period t is a function of the average expected stochastic discount factor in $t+1$ plus the average expectation of the average SDF and price at $t+2$, plus variances. Repeatedly recursive substitution allows us to write prices today as a function of average higher order expectations about future SDFs and variance terms.[5] The model

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[4]The role of higher order beliefs in asset pricing is discussed by Allen et al. (2006), Bacchetta and Van Wincoop (2008), and Makarov and Rytchkov (2012).

[5]Barillas and Nimark (2015) derive more implications of this result, such as the fact that the portion

outlined below is consistent with the assumptions made here, but puts additional structure on the stochastic discount factor; doing so makes it easier to characterize how agents form higher order expectations and how those expectations affect bond prices.

Short rates and higher order expectations. Call x_t be a vector of exogenous factors - "fundamentals" - and conjecture that the one-period risk free rate r_t is

$$r_t = \delta_0 + \delta_x' x_t \tag{2}$$

Assume there are d elements in x_t. Fundamentals follow a VAR(1):

$$x_{t+1} = \mu^P + F^P x_t + C\varepsilon_{t+1} \tag{3}$$

where $\varepsilon_{t+1} \sim N(0, I)$.

Each period, agents observe private signals which are a linear combination of x_t and an idiosyncratic noise component:

$$x_t^j = S x_t + Q \eta_t^j \tag{4}$$

where $\eta_t^j \sim N(0, I)$ is assumed to be independent across agents. For tractability, and in keeping with most of the dispersed information literature, I assume signal precision is the same across all agents, fixed at all times, and common knowledge.

By the no-arbitrage condition (equation (1)), bond prices are related to stochastic discount factors, which themselves are assumed to be a function of fundamentals (x_t). Future stochastic discount factors will be a function of (future) fundamentals. Combined with the fact that bond prices *today* are functions of higher order expectations about stochastic discount factors, the relevant state vector will be the *hierarchy of average higher order expectations* about fundamentals (Nimark (2007)). [6] pth order average expectations are defined

of individuals' expected excess returns due to differences in belief from the cross-sectional average must be orthogonal to public information.

[6]Because of the endogenous price signals and the fact that there are more shocks than signals, there is

recursively as

$$x_t^{(p)} \equiv \int E\left[x_t^{(p-1)} | \Omega_t^j\right] dj$$

and the hierarchy of average order expectations is collected in the vector X_t:

$$X_t \equiv \begin{bmatrix} x_t \\ x_t^1 \\ \vdots \\ x_t^{(p)} \\ \vdots \\ x_t^{(\bar{k})} \end{bmatrix}$$

where $\bar{k}$ is the maximum level of higher order expectation considered.

Conjecture the log bond price is

$$p_t^n = A_n + B_n' X_t + \nu_t^n \tag{5}$$

where ν_t^n is a maturity-specific shock, *i.i.d.* across time and maturities.[7] Further conjecture that X_t follows a VAR(1)

$$X_{t+1} = \mu_X + \mathcal{F} X_t + \mathcal{C} u_{t+1} \tag{6}$$

where u_{t+1} contains all aggregate shocks - the shocks to fundamentals ϵ_t and the vector of price shocks ν_t.

(Log) yields at time t of a zero coupon bond maturing in n periods are defined as $-\frac{1}{n} p_t^n$

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not a closed-form solution for the dispersed information model in general (Huo and Takayama (2014)). In this paper, I use the solution method proposed in Nimark (2007), which solves the model approximately by truncating the state space in terms of orders of beliefs. Huo and Takayama (2014) propose solving this type of model in the frequency domain but also show that in general their method requires approximation when there are endogenous signals. Huo and Pedroni (2017) suggest time-domain methods are faster when the solution method involves numerically finding a fixed point. They find an exact solution for beauty context models with dispersed information and endogenous signals, but their technique is not applicable outside of that context. For this application I set $\bar{k} = 15$. The majority of the weight in bond prices is on the first few orders of expectation; raising the order increases the computational burden substantially without improving the fit of the model.

[7]The role of maturity-specific shocks is to allow for prices to not be fully revealing. As $\sigma_\nu \to 0$, prices become an invertible function of the state and are hence revealing of aggregate information.

where p_t^n is the log price of the bond. Assume bonds up to a finite maturity $\bar{n}$ are traded.[8]
Collect yields in a vector y_t:

$$y_t \equiv \begin{bmatrix} -\frac{1}{2}p_t^2 & \cdots & -\frac{1}{n}p_t^n & \cdots & -\frac{1}{\bar{n}}p_t^{\bar{n}} \end{bmatrix}$$

Assume agents' information sets Ω_t^j include the history of their private signals x_t^j, the short rate r_t and a vector of bond yields out to maturity $\bar{n}$:

$$\Omega_t^j = \left\{ x_t^j, r_t, y_t, \Omega_{t-1}^j \right\} \tag{7}$$

Having conjectured an affine form for bond prices and exogenous information, the signals that agents observe will be an affine function of the state. The filtering problem of an atomistic agent j has the following state-space representation:

$$
\begin{aligned}
X_{t+1} &= \mu_X + \mathcal{F}X_t + \mathcal{C}u_{t+1} \\
\underbrace{\begin{bmatrix} x_t^j \\ r_t \\ y_t \end{bmatrix}}_{z_t^j} &= \mu_z + DX_t + R\begin{bmatrix} u_t \\ \eta_t^j \end{bmatrix}
\end{aligned}
\tag{8}
$$

I assume agents use the Kalman filter to form estimates of the state X_t, which amounts to assuming that agents use Bayes' rule to update their predictions (Harvey (1989)). I also assume agents have observed an infinitely long history of signals, so they use the steady state Kalman filter to make their predictions. This standard assumption avoids the need to keep track of individual signal histories. The matrices $\mathcal{F}, \mathcal{C}$ determine how higher order expectations evolve, which depends on the individual filtering problem of traders and the equilibrium expressions for prices. Prices themselves depend on the evolution of (higher order) expectations. Hence, we first take the bond price equations as given to derive the law of motion, and then show the law of motion is consistent with our conjecture for prices.

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[8]Hilscher et al. (2014) document that the vast majority of Treasury debt currently held by the public has maturity of less than ten years. In the application, I set $\bar{n} = 40$, i.e., 10 years is the maximum traded by agents or used to form forecasts.

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The details of the bond trader's Kalman filtering problem are in appendix B.2. Aggregating across traders implies a fixed point expression for $\mathcal{F}$ and $\mathcal{C}$ (appendix equation (31)).

SDFs and bond prices To derive an expression for prices, I need to explicitly model the stochastic discount factor of bond traders. As is common in affine term structure models, I assume stochastic discount factors are essentially affine (Duffee (2002)). The log SDF is assumed to take the form:

$$m_{t+1}^j = -r_t - \frac{1}{2}\Lambda_t^{j\prime}\Sigma_a\Lambda_t^j - \Lambda_t^{j\prime}a_{t+1}^j \tag{9}$$

In the above expression, $\Lambda_t^{j\prime}$ are (time-varying) market prices of risks to holding bonds, and a_{t+1}^j is the vector of one-period-ahead bond price forecast errors, which have unconditional covariance matrix Σ_a.

$$a_{t+1}^j \equiv \begin{bmatrix} p_{t+1}^1 - E_t^j[p_{t+1}^1] \\ \vdots \\ p_{t+1}^{\bar{n}-1} - E_t^j[p_{t+1}^{\bar{n}-1}] \end{bmatrix} \tag{10}$$

These errors occur because of shocks that were unanticipated by agents. Hence, the vector of forecast errors span the risks that agents must be compensated for.

Assume prices of risk Λ_t^j are an affine function of X_t^j and the vector of maturity shocks:

$$\Lambda_t^j = \Lambda_0 + \Lambda_x X_t^j + \Lambda_\nu E[\nu_t|\Omega_t^j] \tag{11}$$

where X_t^j is are trader j's expectations (from 0 to $\bar{k}$) of the latent factors

$$X_t^j \equiv \begin{bmatrix} x_t^j \\ E_t^j[x_t|\Omega_t^j] \\ \vdots \\ E_t^j[x_t^{(\bar{k})}|\Omega_t^j] \end{bmatrix} \tag{12}$$

As mentioned above, the prices of risk represent additional compensation required for traders

to be willing to hold an additional unit of each type of risk. If Λ_x and Λ_v were zero matrices, risk premia would be constant. If $\Lambda_t^j = \mathbf{0}$, agents would be risk-neutral.

Given the conjectured bond price equation (5): Appendix B shows how to arrive at the following recursive representation of bond prices:

$$
\begin{aligned}
A_{n+1} &= -\delta_0 + A_n + B_n \mu_X + \frac{1}{2} e_n' \Sigma_a e_n - e_n' \Sigma_a \Lambda_0 \quad &(13)\\
B_{n+1}' &= -\delta_X + B_n' \mathcal{F} H - e_n' \Sigma_a \widehat{\Lambda}_x \quad &(14)\\
A_1 &= -\delta_0 \\
B_1 &= -\delta_X'
\end{aligned}
$$

The price of a one-period bond is $p_t^1 = -\delta_0 + [\delta_x, \mathbf{0}] X_t = -r_t$. H is a matrix that selects only higher order expectations terms.[9] e_n' is a selection vector that has 1 in the nth position and zeros elsewhere. $\widehat{\Lambda}_X$ is a normalization of Λ_X.[10]

The state-space representation of prices enables a tractable decomposition of bond yields into average interest rate expectations over the life of the bond, a component driven by higher-order beliefs (the *higher order wedge*) and compensation for risk unrelated to higher order beliefs (the *classical risk premium*). Appendix B.6 contains details of the decomposition.

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[9]More specifically, H is a matrix operator that replaces nth order expectations with $n + 1$-th order expectations and annihilates any orders of expectation greater than $\bar{k}$. This is equivalent to writing prices in terms of a (hypothetical) agent whose SDF is equal to the average.

[10]For comparison, under full information with no maturity-specific shocks, Equations (13) and (14) are replaced by

$$
A_{n+1} = -\delta_0 + A_n + B_n \mu_X - B_n \lambda_0 C + \frac{1}{2} B_n' C C' B_n \quad (15)
$$

$$
B_{n+1} = -\delta_x + B_n F^P - B_n C \lambda_x \quad (16)
$$

3.2 The macroeconomic environment and prices of risk

This section outlines the evolution of the factors x_t that are sources of priced risk in the empirical model. The parameters of the VAR for the factor dynamics are restricted to allow for structural interpretations of the shocks, ensure the model is identified, and to constrain the estimation to economically relevant areas of the parameter space. The assumptions I make are similar to those of Ireland (2015).

3.2.1 Macroeconomic dynamics

Assume short term rates are managed by a central bank that sets an exogenous, time varying, long run inflation target τ_t and then picks a short rate r_t to manage an interest rate gap $g_t^r = r_t - \tau_t$. Define the deviation of inflation from its long run target $g_t^\pi = \pi_t - \tau_t$. Then the evolution of the interest rate "gap" takes the form of a Taylor-type reaction function:

$$g_t^r - g^r = \phi_r(g_{t-1}^r - g^r) + (1 - \phi_r)(\phi_\pi g_t^\pi + \phi_y(g_t^y - g^y) + \phi_v v_t) + \sigma_r \varepsilon_{rt} \qquad (17)$$

In this expression, g_t^y is the output gap. The latent financial risk factor v_t shifts prices of risk Λ_t^j in a manner specified below. I will assume that all time variation in prices of risk comes through movement in this factor. This is consistent with the empirical results in Cochrane and Piazzesi (2008), Dewachter et al. (2014), and Bauer (2016) who all find that a single factor is responsible for nearly all time variation in bond risk premia.[11] Including v_t in the Taylor rule is a simple way to incorporate contemporaneous feedback between financial conditions and the central banks' policy stance. I impose prior restrictions on these parameters. First, I assume that ϕ_v is non-negative.[12] Second, I assume ϕ_r falls between

---

[11] Cochrane and Piazzesi (2008) show that a single "tent shaped" factor extracted from the yield curve explains nearly all time variation in term premia. Dewachter et al. (2014)'s risk factor is identified by a similar assumption to that of Ireland (2015) and is highly correlated with the Cochrane-Piazzesi factor. Bauer (2016) use Bayesian methods to estimate a Gaussian term structure model and finds evidence for strong zero restrictions which imply only changes in the "slope" factor affect term premia.

[12] While in principle unnecessary for identification, this restriction is consistent with the idea that the

zero and 1. Finally, I assume ϕ_π and ϕ_y are both positive.

The long-run inflation target is assumed to follow an $AR(1)$ process:

$$\tau_t = (1 - \rho_{\tau\tau})\tau + \rho_{\tau\tau}\tau_{t-1} + \sigma_\tau \varepsilon_{\tau t} \tag{18}$$

with $\rho_{\tau\tau} \in (0,1)$.[13]

Collecting the factors in x_t:

$$x_t = \begin{bmatrix} g_t^r \\ g_t^\pi \\ g_t^y \\ \tau_t \\ v_t \end{bmatrix} \tag{19}$$

they can be written in matrix form:

$$P_0 x_t = \mu_x + P_1 x_{t-1} + \Sigma_0 \varepsilon_t \tag{20}$$

Exact expressions for $P_0, \mu_x, P_1, \Sigma_0$ are shown in appendix B.4. Left multiplying by P_0^{-1} yields (3). After a normalization of one covariance matrix parameter, the VAR is exactly identified. I calibrate $\sigma_v = 0.01$.

Restrictions on prices of risk. The matrices governing the mapping of factors into prices of risk shown in (9) and (11) are high-dimensional. As Bauer (2016) notes, absent restrictions on the prices of risk, the estimation does not take into account cross-sectional information in the yield curve. Accordingly, I incorporate two sets of restrictions. First, I follow Ireland (2015) in imposing that, under *full* information, changes in prices of risk are driven by entirely by changes in v_t, and that v_t is not itself a source of priced risk.Second,

central bank has raised rates in response to an increase in risk premia. McCallum (2005) suggests a Taylor rule with smoothing and a reaction to the term spread - itself affected by a possibly time varying term premium - is consistent with a negative slope coefficient in Campbell and Shiller (1991) regressions.

[13]Stationarity is assumed for two, related, technical reasons. The first is that interest rate processes that contain a unit root will leave long-run yields undefined. The second, related issue, is the stationarity of asset prices helps ensure that approximation error caused by truncating $\bar{k}$ can be made arbitrarily small (Nimark (2007)).

I follow Barillas and Nimark (2015) in restricting Λ_t^j to nest the full information version of the model without maturity shocks. This means that the same number of parameters govern prices of risk in the full and dispersed information models.[14] Details of how these restrictions are implemented are shown in appendix B.5.

3.3 Signals

The last step is to specify agents' agents' idiosyncratic signal structure. I do not formally model the information choice of traders but impose an exogenous information structure.[15] I assume prices are observed without error, but individuals' observations of the non-price factors driving prices of risk are subject to idiosyncratic noise that is uncorrelated across variables. Recalling (4), I assume bond traders observe the short rate and separate signals about inflation and the long-run inflation target. To summarize:

$$
x_t^j = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} x_t + \begin{bmatrix} 0 & 0 & 0 & 0 \\ \widetilde{\sigma}_\pi & 0 & 0 & 0 \\ 0 & \widetilde{\sigma}_y & 0 & 0 \\ 0 & 0 & \widetilde{\sigma}_\tau & 0 \\ 0 & 0 & 0 & \widetilde{\sigma}_v \end{bmatrix} \begin{bmatrix} \widetilde{e}_t^\pi \\ \widetilde{e}_t^y \\ \widetilde{e}_t^\tau \\ \widetilde{e}_t^v \end{bmatrix} \tag{21}
$$

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[14]Like Barillas and Nimark (2015) I also assume the maturity specific shocks shocks have the same standard deviation across yields, although the shocks to each yield are independent.

[15]Exogenous information keeps the model tractable enough to allow for likelihood based estimation. The downside is vulnerability to a Lucas-critique-like argument that the allocation of attention is not invariant to policy changes, and the model does not let the precision of signals vary over the business cycle, as it might in a model where agents optimally (re)allocate attention. The advantage is this allows estimation of the precision of traders' information that is consistent with asset price movements over the sample.

4 Solution and Estimation

4.1 Solution

The solution to the model is a fixed point of the bond pricing terms (A_n, B_n) and agents' beliefs. In particular, we need to find a fixed point between the price recursions, equations (13) and (14), the mean-square error matrix for state forecast error (equation (29) in the appendix), and the law of motion for the hierarchy of average higher-order expectations (equation (31) in the appendix). The precise numerical procedure for finding a fixed point is detailed in appendix C.

4.2 Econometric Model and Data

The model period is a quarter and the estimation runs from Q4:1971-Q4:2007. The end date is chosen prior to the zero lower bound period because the linear model does not respect the ZLB constraint.[16] I take data on (non-annualized) zero coupon yields from the yield curve estimates in Gurkaynak et al. (2007), averaged over the quarter. In the econometric model, I use the the short rate (assumed to be the Federal Funds Rate, as in Piazzesi et al. (2013)), and rates on 1,2,3,4, 5 year and 10 year bonds.[17]

To identify agents' beliefs and the macroeconomic dynamics, I use data on the output gap (calculated as the log difference between real GDP and its HP filtered trend using a smoothing parameter of 16,000)), inflation and inflation forecasts (as measured by log changes in the GDP deflator), and treat τ_t and v_t as latent process.

---

[16]The recent literature on shadow rates - for example Wu and Xia (2014) and Bauer and Rudebusch (2016) - emphasizes that dynamic term structure models perform poorly when the zero lower bound is not taken into account. Moreover, the zero lower bound introduces a nonlinearity in the signal structure which the conventional Kalman filter does not take into account. Estimating shadow rate models involves either discretization of the state space or simulation-based methods which are computationally infeasible in this dispersed information setting.

[17]Note agents are assumed to observe the whole yield curve, not just this subset.

I use the cross-section of one and four quarter ahead forecasts from the Survey of Professional Forecasters in the estimation. The advantage of inflation forecasts is that they are available for the entire sample period with a relatively high response rate. Moreover, inflation forecasts in the SPF are quite accurate on average, which means my choice of data does not automatically favor sizable information frictions. Individual survey responses are treated as a noisy indicator of the average expectation, where the extent of the noise is pinned down by the model-implied cross-section of expectation around the first-order average expectation. This matrix can be calculated using the Kalman filtering problem of individual agents (see appendix B.2). Because the number of respondents to the SPF has varied over time, the number of observables at different times is time varying. Accordingly, the Kalman filter equations used to estimate the likelihood of the model are time varying. Assuming there are m_t^1 respondents to the 1-period ahead question and m_t^4 to the four-period ahead question in the SPF at time t, the state space system for estimation is

$$
\begin{aligned}
X_t &= \mu_X + \mathcal{F}X_{t-1} + [\mathcal{C}, \mathbf{0}_{d(\bar{k}+1)\times m_t^1+m_t^4}]\bar{u}_t \\
\bar{u}_t &\sim N(0,1) \text{ with dimension } (d + (\bar{n} - 1) + m_t^1 + m_t^4) \times 1 \\
\bar{z}_t &= \mu_{\bar{z},t} + \bar{D}_t X_t + \bar{R}_t \bar{u}_t
\end{aligned}
\tag{22}
$$

where in particular $\mu_{\bar{z},t}$, $\bar{D}_t$ and $\bar{R}_t$ vary in size to account for missing observations. The matrices are reported in appendix D.

For the full information version of the model, I treat forecasts as if they are observations of the rational expectations forecast with $i.i.d.$ error. I allow each forecast horizon to have a different error variance. I also treat each individual bond yield as if it were observed with maturity-specific econometric error. Conceptually, these errors are distinct from the errors in the dispersed information version. In the dispersed information model, the "noise" in forecasts is pinned down by the model-consistent state mean square error matrix. As discussed earlier, the maturity specific shocks are a risk faced by traders in the model, rather than being econometric noise in the empirical model.

20

I estimate the model via Bayesian methods. In particular, I use a Metropolis-Hastings Markov Chain Monte Carlo procedure to estimate the model parameters. Because the model has a large number of parameters and is computationally burdensome to solve, I use somewhat informative priors on macroeconomic variables to focus on reasonable areas of the parameter space. As noted earlier, I restrict $\phi_v \geq 0$. I also place some informative prior restrictions on VAR parameters. I impose that ρ_{yv} is non-positive, which implies that all else equal, greater risk premia are contractionary. This is consistent with most general equilibrium models with financial frictions. For similar reasons, I impose a slightly informative prior that for ρ_{yr} that is centered around -1, while still allowing the estimation to explore regions of the parameter space where this restriction does not hold. Finally, I follow Ireland in calibrating $\rho_{\tau\tau} = 0.999$. Prior distributions are reported in appendix G.

I follow Ireland (2015) in imposing the identifying restriction that $\lambda_\pi^x < 0$ to pin down how the latent process v_t affects risk premia. To ensure long-run bond prices are well defined the estimation imposes that physical and risk-neutral dynamics of bonds are stationary under full information. This implies only accepting parameter draws such that the maximum eigenvalues of F^P and $F^P - C\lambda_x$ are less than one in modulus.

To summarize identification: The macroeconomic VAR is exactly identified by impact restrictions on the exogenous processes. Because agents are assumed to have rational expectations, their beliefs about the macroeconomic VAR are correct. Hence, any mistakes in their inference come from noise in their signals, and the extent to which they *disagree* about the state depends on the noise in their private signals. If private signals were perfectly revealing, or arbitrarily noisy, then they would receive no weight and agents would not disagree because they would condition on common information. To the extent agents disagree, it must be because their private signals are sufficiently informative to receive some weight. Hence, the cross-section of inflation forecasts is informative about private signal noise because it reveals the extent of disagreement. Agents' beliefs must also be consistent with bond prices, so

yields are also informative about agents' beliefs. Given a belief process for macroeconomic variables, agents' beliefs about expected short rates are also pinned down; what remains in yields has a component that covaries with macroeconomic variables (pinning down the time-varying part of prices of risk) and an average component (the constant part of prices of risk). Prior information is also somewhat informative macroeconomic parameters.

I run separate MCMC chains in parallel for each model. For the full information model, each chain is of length 400,000; I discard the first 10% of each chain and subsequently analyze every 1000th draw. The DI model is much more computationally intensive; the results reported here are based on 5 chains of length $23,000$ each. I drop the first 50% of each chain (because it takes longer to stabilize) and use every 100th draw.

5   Parameter Estimates and Impulse Responses

Here I report the results of the estimation for the dispersed information model.

5.1   Parameter estimates

Parameter estimates across chains, and posterior credible sets are reported in appendix table (6).[18] In terms of macroeconomic dynamics, the full and dispersed information models have relatively similar parameter estimates. This is, unsurprising, as the model does not allow for direct feedback from the inference problem of agents to the macroeconomy. Most of the macro-VAR parameter estimates are basically in line with the results in Ireland (2015), although estimates of the prices of risk differ. While some of this is likely attributable to differences in samples, the full information estimates of those parameters have a high degree of dispersion, as do the parameters governing the covariance of the non-financial factors with the risk factor v. Taking the full and dispersed information parameter estimates together, it

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[18]Full information parameter estimates are shown in appendix I.

appears that it is difficult to separately identify the prices of risk terms, and the covariances that govern changes in risk.

The key parameters of interest govern the informational quality of agents (the bottom five rows of the table). The relatively small value for σ_ν implies, all else equal, that prices move mostly due to (higher order beliefs about) fundamentals rather than large direct shocks to prices. This suggests the model prefers to endogenously explain movements in yields.

Agents' idiosyncratic noise appears somewhat large in absolute terms. However, this does not necessarily imply agents' beliefs are inaccurate, because they understand the structure of the economy. For example, traders know an unanticipated increase in inflation is correlated with unanticipated increases in output ($\sigma_{y\pi} > 0$), and that higher inflation today usually depresses growth in the future ($\rho_{y\pi} < 0$). Moreover, agents learn from prices, which aggregate information. We cannot conclude simply from the parameter estimates that agents have inaccurate beliefs. All else equal, noisier private signals receive less weight.

5.2   Impulse Responses

To demonstrate some of the information mechanisms at play, I plot impulse responses for fundamentals, the first three orders of expectation about fundamentals, inflation forecasts, and prices for a subset of the model shocks. The impulse responses discussed in this section are shown for the posterior mode for clarity. The complete set of impulse responses for the dispersed information ATSM, including posterior credible sets, shown in appendix H.1. [19]

The fundamental impulse responses to one-standard deviation shock to the monetary policy rule are shown in figure 2. The top row displays the responses of the fundamental factors, while the subsequent rows show increasing higher-order beliefs about those variables (with inflation expectations in the far right column). For inflation and interest rates, the

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[19]The complete set of full information impulse responses are omitted for space reasons, but are available on request.

responses are in terms of annualized percentage points; the output gap is in percentage points, and "risk" is scaled up by 100. As expected, a shock to the short rate causes inflation and output to fall over the course of several years.
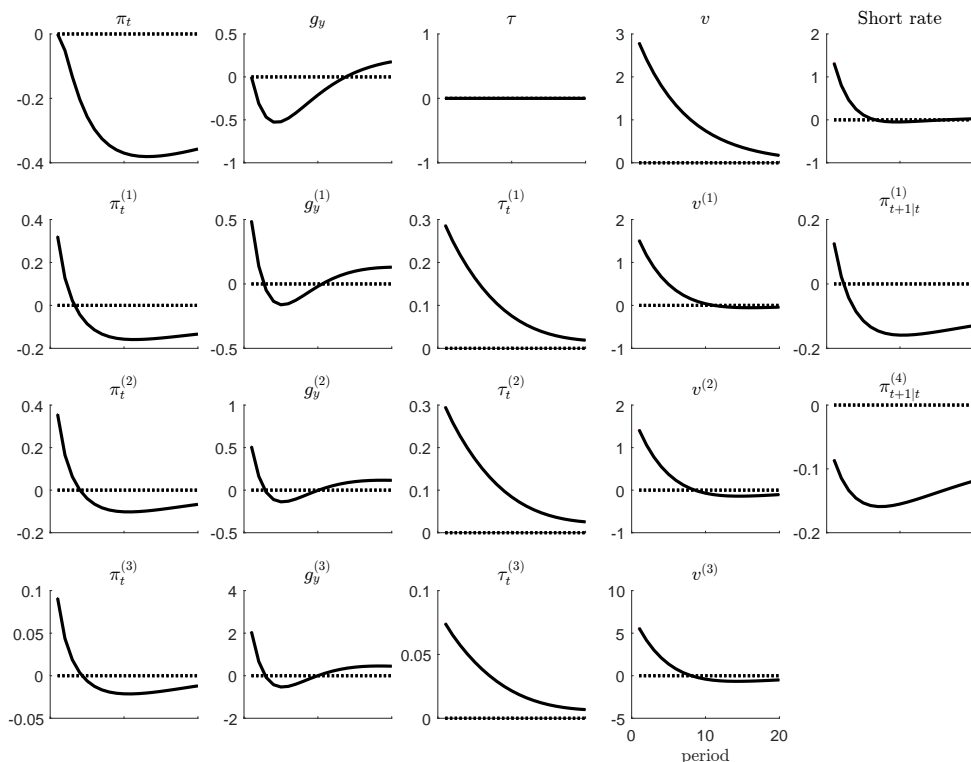
Rate shock: Macroeconomic Responses

Figure 2: Response of non-financial variables to monetary policy rule shock, dispersed information model

The impulse responses illustrate the identification problem faced by agents in the model. Agents observe the short rate has risen but also know this could be caused by any of the fundamental shocks. Because they are unable to discern the origin of the change, they place some posterior weight on the possibility that both inflation and the inflation target

are elevated.[20] Agents persistently misattribute the cause of the increase in short rates to changes in the inflation target. They further believe others believe the inflation target has risen (second order expectations are similar to first order), but third order beliefs increase by less. This implies traders' beliefs, in addition to being imperfect, are dispersed - on average, they believe that others do not share their beliefs, especially about others' beliefs. Over time, as they observe the evolution of prices and their noisy signals about macroeconomic dynamics, their beliefs approach the true impulse responses (top row). In short, optimal inference in the model is characterized by mistaken beliefs about the origins of shocks and divergence of average beliefs from higher order beliefs.

Interestingly, dispersion of beliefs after an interest rate shock does not have a large direct effect on yields (figure 3). The overall response of yields to the shock are shown in the first row. Subsequent rows show the decomposition into rate expectations, "classical" risk premia, and the higher order wedge (as described in appendix B.6). Average rate expectations (row 2) are elevated as a result of the shock, which explain nearly all of the increase in yields, even at the long end of the yield curve. In other words, agents may not know *why* rates have increased, but everyone agrees the path of short rates will be persistently elevated. This is driven by beliefs about the inflation target, which raises the expected path of short rates. Classical risk premia (row 3) and the higher order wedge (row 4) barely move as a result of the shock.

Because the path of expected short rates does not adjust as quickly as it would under full information, long term yields rise more after a rate shock and remain elevated. In other words, the inference problem of agents in and of itself matters for the assessment of financial fluctuations. As will be shown in section 7, once we account for the slow adjustment of expectations, yields are mostly attributed to the expectations hypothesis component.

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[20]The rise in inflation expectations may seem puzzling. However, it is consistent with the sign-restricted VAR results of Melosi (2017) and Struby (2018); in US data, identified monetary policy shocks cause inflation expectations to rise on impact, perhaps because of signaling effects.
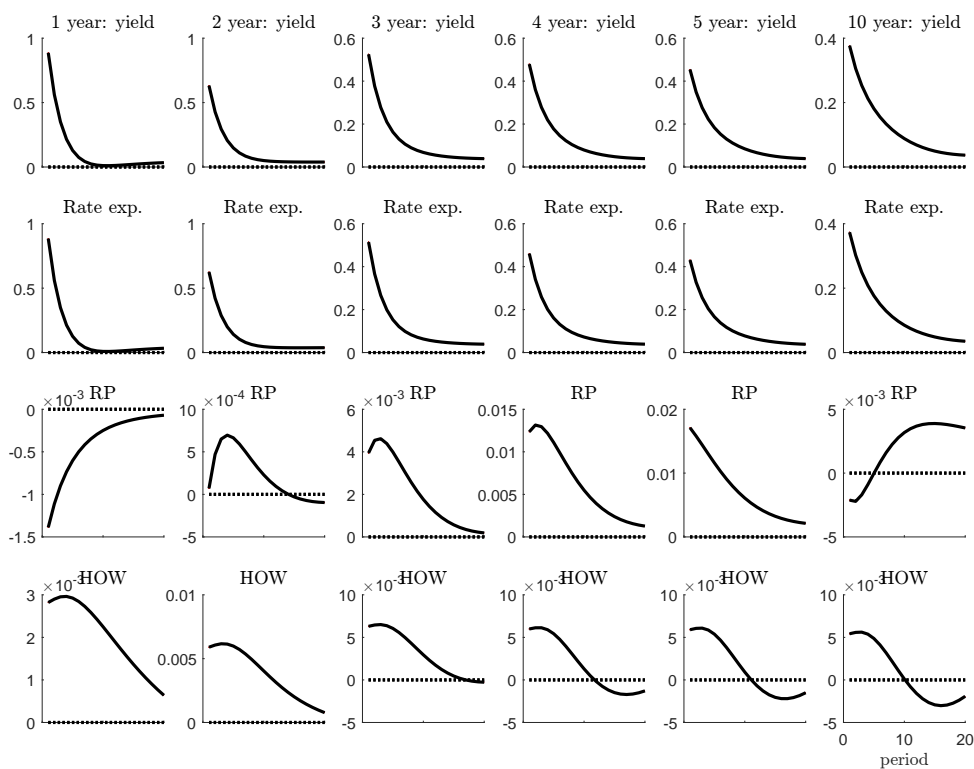
Rate shock: Yield Responses

Figure 3: Response of financial variables to monetary policy rule shock, dispersed information

A similar set of impulse responses for a one standard deviation increase in the inflation target τ are shown in figures 4 and 5. Movements in the inflation target cause level shifts in the yield curve by persistently raising short rates.[21] Agents are slow to adjust the shock, so the level shift is gradual, rather than immediate, but the shock to the inflation target raises yields across the board by approximately the same amount over the course of several years. This is similar to the role it plays in the full information model (included in appendix I)

What is interesting is the difference between fundamentals (the top line in figure 4) and higher order beliefs about those fundamentals. As in the full information model, a

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[21]Recall that the inflation target is the most persistent shock, with its autoregressive component calibrated to $\rho_{\tau\tau} = 0.999$.

higher inflation target is associated with a temporary expansion in output. However, agents observing higher rates, accompanied by upward movements in inflation and risk, actually believe that output rises initially, falls over the medium term, and rises again. Higher order believes follow this pattern, although third-order expectations move more dramatically.
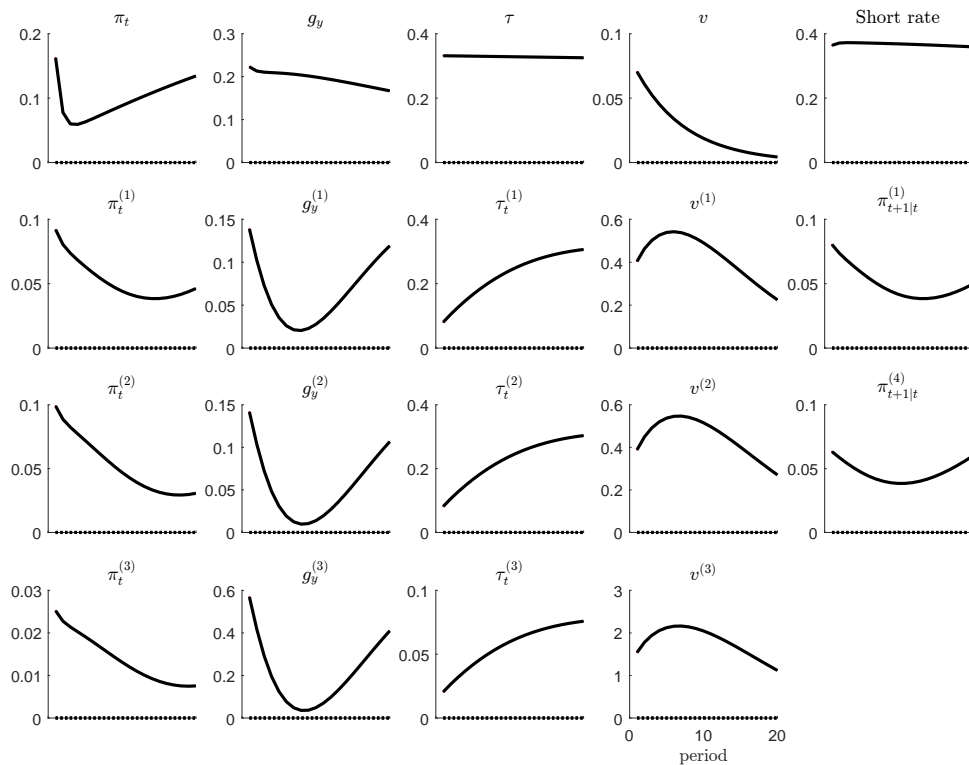
Target shock: Macroeconomic Responses

Figure 4: Response of non-financial variables to inflation target shock, dispersed information

"Risk shocks" (shocks to v) are shown in figures 6 and 7. The effect of risk shocks on macroeconomic variables and asset prices stand in contrast to the risk shocks in Ireland (2015). In his paper, the co-movements brought on by risk shocks are qualitatively similar to those of a monetary policy shock, albeit without a "price puzzle." By contrast, impulses to v_t in both the full- and dispersed-information versions of the model estimated here have
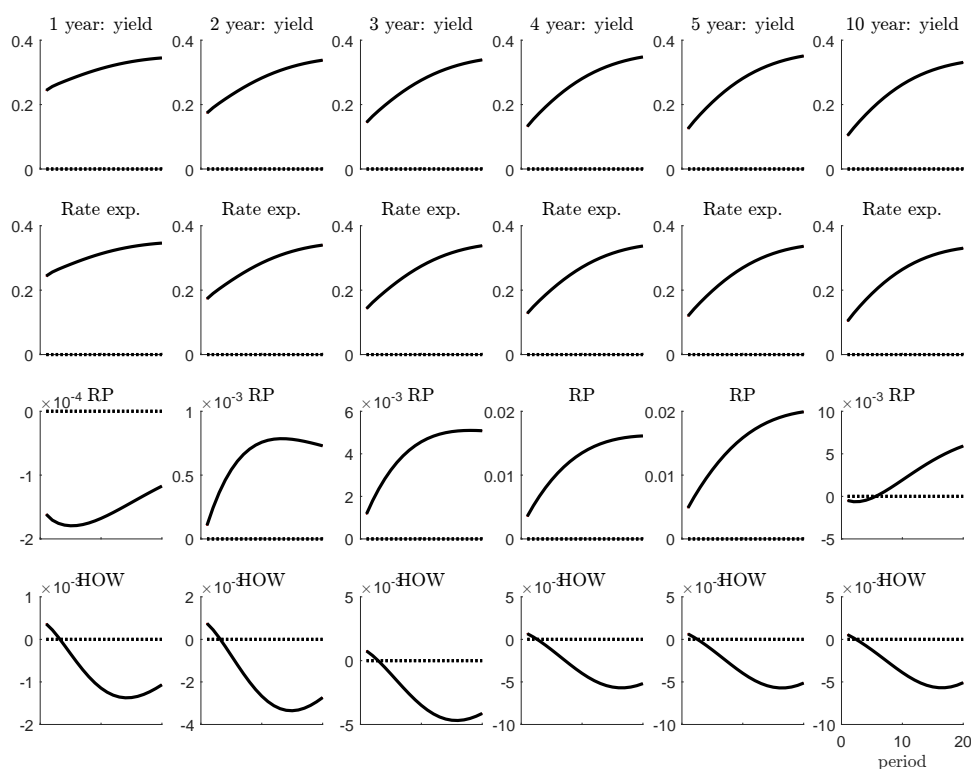
Target shock: Yield Responses

Figure 5: Response of financial variables to inflation target shock, dispersed information

nearly no direct effect on output, but depress inflation, and the reduction in inflation leads output to grow over time. Since this holds for both the full- and dispersed-information models, it is not a result of the information assumption.[22]

In terms of asset prices, risk shocks increase risk premia more than monetary policy shocks. More importantly, risk premia move much less in the dispersed information model

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[22] Two possibilities for differences in the dynamic behavior for v_t between these results and those of Ireland (2015) present themselves. One is that this is driven by differences in sample, particularly, the difference in sample period. One other sample difference is that I include inflation forecasts. Removing those forecasts from the dataset does not qualitatively change the impulse responses at the posterior mode of the full information model. The second possibility, alluded to earlier, is that the risk parameters and the prices of risk are not particularly well identified. Both of these explanations suggest stronger prior information might "smooth out" the posterior and make the impulse responses to risk shocks more strongly resemble those of Ireland. In the absence of strong priors, these dynamics appear to be what the data prefers.
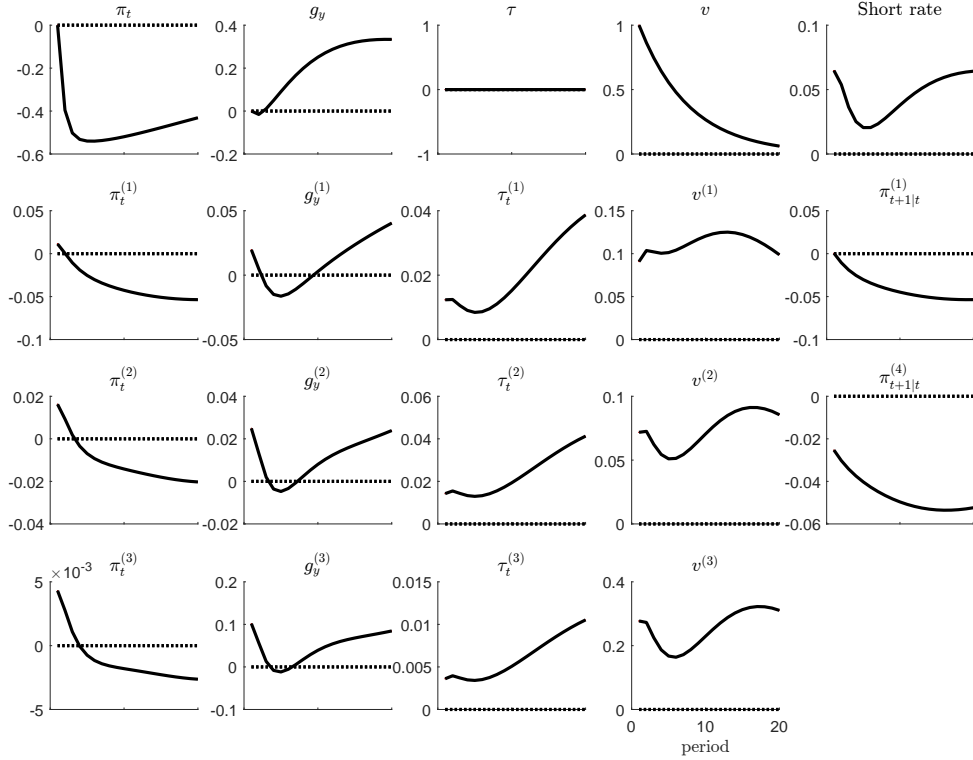
Risk shock: Macroeconomic Responses

Figure 6: Response of non-financial variables to risk shock, dispersed information

than under full information. From the point of view of the dispersed information model, the full-information rational expectations risk premia conflates the difference between full information average expectations and actual average expectations, the effect of higher-order expectations, and "classical" risk premia, which are the part of the counterfactual consensus price unexplained by average rate expectations. Since the parameter estimates imply different expected short rates, the full information risk premia appears to be largely driven by the assumption of full information rational expectations.

Other impulse responses are shown in appendix H.1. The macroeconomic implications are as expected. Shocks to the output gap induce positive comovement in inflation, output,
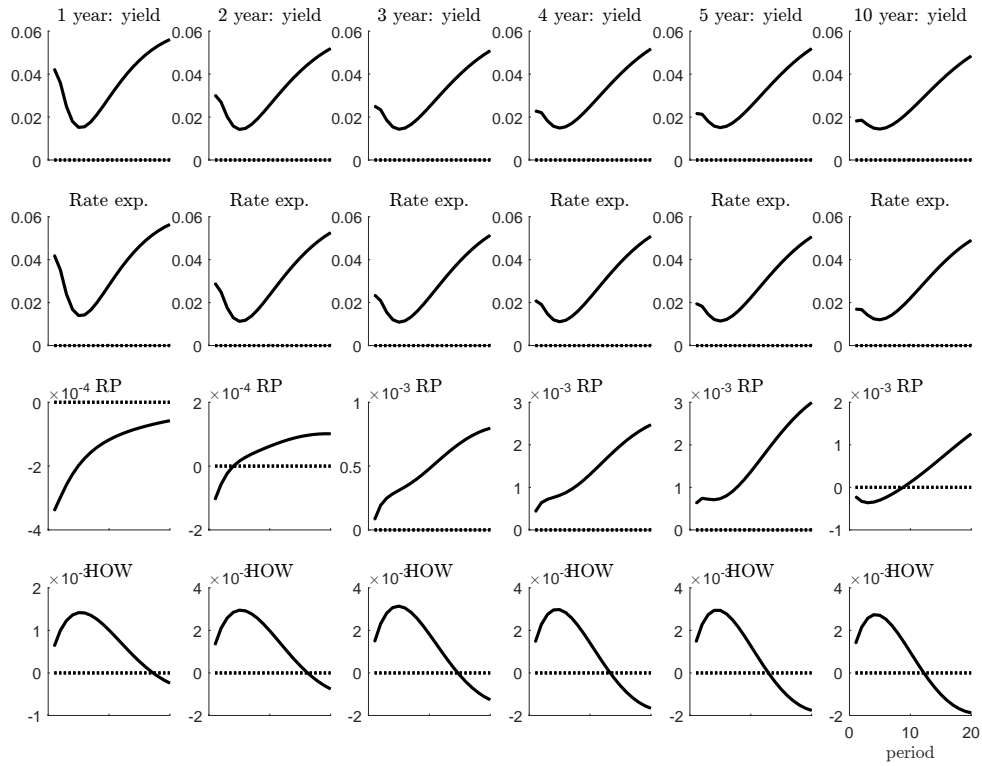
Risk shock: Yield Responses

Figure 7: Response of financial variables to risk shock, dispersed information

and interest rates - in a sense they are similar to demand shocks in DSGE models. Inflation shocks raise output on impact but lower it over the medium term. Apart from the initial (positive) change in output, they somewhat resemble cost-push shocks to the Phillips curve in New Keynesian models.

6   What do traders learn from?

In this section, I characterize the informativeness of agents' signals. In most models of dispersed information, agents are assumed to learn only from idiosyncratic signals about

fundamentals. Since agents' noisy signals are true on average, it is possible that asset prices clean out idiosyncratic noise and agents are able to determine the true realization of fundamentals. And, because prices reflect the beliefs of agents, it may be possible that yields do not contain any information that agents do not already know. However, the estimates imply that despite abundant common information, agents' information is imperfect and dispersed. To preview the results, agents learn about half of what they know about the output and inflation gaps from their private signals, and learn much less about policy or the financial risk factor. The rest of their information about fundamentals comes from prices. Moreover, the majority of traders' information about the beliefs of others also comes from observing prices. The most important price signal appears to be the policy rate of the central bank, which is also the yield on a bond that matures in one period. The short rate is informative about fundamentals, and because everyone knows that everyone learns about fundamentals from this particular signal, it is also informative about higher-order beliefs.

The approach I use to understand the informativeness of prices is drawn from information theory. In particular, agents' posterior uncertainty can be characterized in terms of entropy, which can be thought of as the average number of binary signals needed to fully describe the outcome of a random variable.[23] We can characterize how much agents learn from signals about a particular variable in terms of the reduction in entropy after observing those signals (see appendix F for details). Adapting a measure used in Melosi (2017), I examine how informed agents are after viewing a counterfactually limited subset of signals *relative* to to how informed they would be if I let them use the complete set. In other words, I calculate how informed they are after observing all of their private signals and the yield curve. I then calculate how informed they would have been under a counterfactual subset of signals. Since on average additional information must (weakly) reduce uncertainty, we can think of the

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[23]The entropy-based measure of signal informativeness I use is also used in the rational inattention literature initiated by Sims (2003) to describe the constraint on agents' information processing capacity.

Table 1: Reduction in uncertainty about fundamentals (columns) coming from observing a single signal (rows).

Signal($\downarrow$), fundamental $\rightarrow$	π	g^y	τ	v
r_t	0.42	0.45	0.97	0.64
π^j	0.03	0.03	0.12	0.00
g^{y^j}	0.03	0.07	0.04	0.00
τ^j	0.00	0.00	0.09	0.00
v^j	0.37	0.33	0.03	0.24

reduction in entropy coming from the subset of signals as the fraction of total information that could have come from that set. If the reduction in entropy were zero, it would imply there was no information in those signals. The advantage of this measure is that it respects both that agents' inference is optimal (by assuming that they do the best they can with whatever signals they have) and signals may be redundant.

Table 1 shows the relative reduction in uncertainty about macroeconomic fundamentals variables (columns) from observing the short rate (first row) or a single private signal (remaining rows). These represents extreme constraints on agents' information. The second row, for instance, suggests, that very little (around 3%) of the information traders have about inflation comes from their inflation signal in particular (second row, first column).[24] Three features of the table stand out. First, individual private signals are not terribly informative in general. Second, as to be expected from the fact that agents understand the structure of the model, signals are informative not just about their own realization but about the realizations of other variables; for example, knowing something about inflation tells you something about the output gap. This feature of the world is ignored in most exogenous information models because they typically assume agents learn about independent exogenous processes.

Indeed, observing only the short rate would give you more information about fundamentals than observing any *individual* noisy signal. This is likely for two related reasons. First,

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[24]The columns will not generally sum to 1 because some information is redundant between signals and because yields are also informative.

Table 2: Reduction in posterior uncertainty about about fundamentals and higher order beliefs from observing only private signals

	g^r	g^π	g^y	τ	v
x_t	0.02	0.41	0.41	0.16	0.24
$x_t^{(1)}$	0.01	0.09	0.08	0.09	0.02
$x_t^{(2)}$	0.02	0.12	0.11	0.08	0.03
$x_t^{(3)}$	0.04	0.12	0.12	0.08	0.04

the short rate depends directly on the contemporaneous realization of all fundamentals. Second, it is observed without error. Despite the fact that agents are unable to perfectly identify which fundamental moved the short rate, they do know that noise does not factor into their observation; any movement in the short rate is important.

Table 2 shows (relative to the benchmark with price signals) how much agents' posterior uncertainty is reduced by conditioning only on their four private signals. Here, I switch to considering the risks agents face (i.e., leaving rates and inflation in terms of their gaps) rather than realizations.

As the first row of the table reveals, agents' private signals are most informative about the inflation and output gaps. Agents get just under half of their information about the macroeconomy from their private signals. They can learn very little about risk and the implicit inflation target from observing their idiosyncratic signals and almost nothing about the rate *gap* $g_t^r \equiv r_t - \tau_t$ (recall they are assumed to not observe the short rate in this counterfactual). The remaining lines show how much of their information about (higher-order) expectations come from private signals. Since private signals are about the true realization of variables, rather than higher-order beliefs about those variables, they are only indirect signals about higher order beliefs.

Another way of thinking about the results in table 2 is "what are price signals informative about?" It turns out that the majority of information agents have about the financial risk factor (v) and monetary policy (summarized by g^r and τ) comes from observing price signals,

including the short rate. Nearly all of their information about the first three orders of expectations is encoded in price signals (rows 2-4). Yield curve variables may not be fully informative about fundamentals or the beliefs of others, but the vast majority of information traders have about the latter seems to come from prices.

This result has two immediate implications. First, it validates thinking of the yield curve as a summary measure of what bond traders believe. Indeed, the model implies that the best *bond traders* can do to understand what others believe is by combining their understanding of how expectations are determined with the prices they observe. Since prices depend mostly on higher-order beliefs, prices are useful to bond traders even though they aren't fully informative about fundamentals. Second, the results caution against ignoring the informativeness of prices - agents may have very inaccurate signals on average, but the ability to learn from prices makes that less consequential. This matters directly for models featuring dispersed information. An econometrician calibrating the informativeness of private signals using only forecasts while ignoring the role of learning from prices would incorrectly conclude that private signals must be quite accurate.

The estimates imply most of what traders learn can be found by combining of their private signals and the short rate. The results of this counterfactual are shown in table 3. Effectively all of what they know about monetary policy and risk comes from their private signals plus the policy rate, and 75% or more of what they know about the first three orders of expectation can be extracted without using bonds with a maturity of greater than one quarter.

This result adds to recent evidence, such as that of Tang (2013) and Melosi (2017) that the Federal Reserve's policy instrument is an important signal. It tells observers a great deal about macroeconomic fundamentals and policy risks. And because it is a public signal that evidently contains a lot of what agents know about fundamentals, it plays an outsized role in market participants' higher order beliefs (along the lines of Morris and

Table 3: Reduction in posterior uncertainty about fundamentals and higher order beliefs from observing private signals and r_t

	g^r	g^π	g^y	τ	v
x_t	0.98	0.85	0.88	0.99	0.93
$x_t^{(1)}$	0.86	0.72	0.72	0.92	0.90
$x_t^{(2)}$	0.85	0.81	0.78	0.90	0.92
$x_t^{(3)}$	0.86	0.85	0.84	0.90	0.94

Table 4: Reduction in posterior uncertainty about fundamentals and higher order beliefs from observing private signals and ten year yield

	g^r	g^π	g^y	τ	v
x_t	0.23	0.82	0.76	0.94	0.60
$x_t^{(1)}$	0.10	0.68	0.55	0.70	0.36
$x_t^{(2)}$	0.12	0.77	0.65	0.66	0.32
$x_t^{(3)}$	0.17	0.82	0.72	0.63	0.30

Shin (2002)). Assuming agents learn only from private signals and the policy rate - the information assumption of Melosi (2017) and Kohlhas (2015) - is a fair approximation of what bond traders appear to learn from.

To emphasize the fact that the policy rate is somewhat special, table 4 shows a counterfactual where agents learn from their private signals and ten year yields rather than the federal funds rate. Ten year yields are informative about both fundamentals and higher-order beliefs above and beyond private signals, but they are less informative than the policy rate. This is especially true about the current policy gap (the first column) and the risk factor v (the last column).

Why are ten years yields less informative? The price of a ten year bond is determined not just by fundamentals (the short rate), but also higher order beliefs about the evolution of fundamentals over the next ten years, plus the maturity-specific shock. The fact that the bond is of longer maturity means that (increasingly) higher order beliefs play a greater role in its price. The fact that shocks to fundamentals are transitory, higher-order beliefs play a bigger role in prices, and that bond prices are affected by maturity specific shocks, imply

they will be less informative about current fundamentals. If, for instance, bond prices were not affected by noise, then they would be fully revealing of fundamentals and agents would not need to rely on the short rate as a source of information.

These results come with a few caveats. First, in keeping with the majority of the literature, the model is constructed specifically to price a single type of asset. Other types of assets may be informative about a different set of macroeconomic or idiosyncratic risks. Information from other asset types would be captured by the precision of private signals. Second, the results of this section are partial equilibrium in the sense that the model does not allow for direct feedback from expectations to macroeconomic aggregates. But from the point of view of an atomistic agent, macroeconomic aggregates are exogenous processes and the precise role of information in generating aggregate fluctuations should not matter.

7 Decomposing (Higher Order Expectations in) the Yield Curve

Despite the abundance of public signals, non-trivial dispersion of higher order beliefs persists in the model. A natural question is what *direct* effect this dispersion of belief has had on prices, and more generally what the model attributes variation in bond yields to. In this section, I use estimates of the underlying higher-order beliefs to decompose prices as outlined in appendix B.6.[25] I use this to answer two questions: (1) What does the model attribute changes in bond yields to - changes in rate expectations, "classical" risk premia, or higher-order beliefs? (2) Which higher-order beliefs matter for prices? Briefly, the answer to the first question is that (slowly adjusting) rate expectations play the largest role in determining

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[25]The results here are based on Kalman filtered estimates of the state, which can be thought of as inefficient estimates of the underlying hierarchy of higher-order expectations X_t. Kalman smoothing, which takes account of the whole sample to derive estimates, presents numerical problems because the one-step ahead state forecast error matrix is numerically ill-conditioned. The filtered estimates are closest to what the Kalman smoother would imply at the end of the sample.

yields at all horizons. Classical risk premia are nearly constant for bonds at all maturities, but the importance of the higher order wedge increases in the maturity of the bond. As for the second higher order beliefs about monetary policy variables - the rate gap g^r and the inflation target τ - drive most of the time variation in the wedge.

Informally, we can think about yields as be being driven by a part that is rate expectations and a residual. The information assumption implies the path of rate expectations, and thus determine the "expectations hypothesis" part of bond yields. Combined with our assumption of no-arbitrage and the way risk is priced, full information rational expectations implies the residual is the risk premium. However, as the decomposition outlined in appendix B.6 shows, under dispersed information the residual can be interpreted as the sum of the higher-order wedge and the gap between average expected short rates and the price that would obtain if agents counterfactually held common beliefs. The residual in the full and dispersed information models may also be different because they imply different short rate forecasts.

Although one could focus on bonds of any maturity, here I focus on ten year yields.[26] The three-way decomposition is shown in figure 8. Comparing the top two panels, it is clear that the model attributes the majority of movement in bond yields to rate expectations. In other words, accounting for agents' subjective rate expectations makes the implied premium for investing in long term bonds less volatile. That premium is divided between the "classical" premium and the higher order wedge; they are of roughly equal magnitudes, but the former is close to constant while the wedge varies over time. The reduced importance of compensation for risk in determining bond yields is qualitatively consistent with Piazzesi et al. (2013), who use a very different methodology to arrive at this conclusion.

This result implies at least part of the dramatic failure of the expectations hypothesis is attributable to assuming agents' expectations are based on full information. Accounting for the fact that agents' subjective forecasts may be different from the underlying full information

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[26]The results for other maturities are found in the appendix H.3.
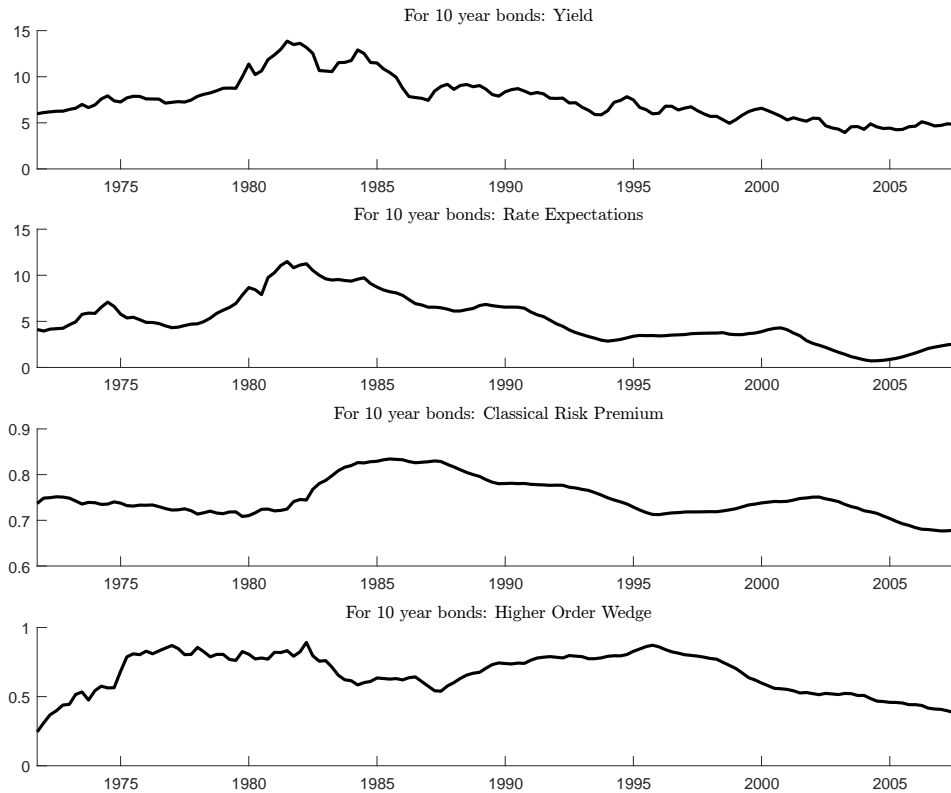
37

Figure 8: Decomposition of 10 year yields, dispersed information

forecast means that volatile time varying risk premia are not needed to explain movements in long term yields. The remaining premium for holding long term debt is partially about time varying compensation for risk (the classical risk premium) that is unrelated to disagreement. However, the the larger, time varying portion is attributable to the failure of consensus - that is, the fact that agents believe others have different beliefs. Both classical risk premia and the higher order wedge rose during the Volcker disinflation and declined afterwards.

Why did risk premia and the higher order wedge decline over time? The model restrictions imply a decline in the classical risk premium must be attributable to a decline in v_t over time. The inflation target variable τ_t is also falling over this period and since the estimated

results imply $\sigma_{v\tau} > 0$ the decline in the inflation target appears to have driven the decline in risk.[27] We can also examine the role of higher order beliefs about these variables in determining yields. In figure 9, I show the higher-order wedge decomposed into the contribution from higher-order beliefs about fundamentals. The decomposition reveals that the model attributes growth in the higher order wedge to an increasing role for higher order beliefs about the inflation target. A smaller contribution comes from higher order beliefs about the rate gap $g^r = r_t - \tau_t$. Since r_t is commonly observed, this means that overall *policy uncertainty* contributes the most time variation to the wedge, at least for ten year yields. This is (partially) counterbalanced by higher order beliefs about the risk variable, which grew in the late 1970s and 80s but fell afterwards.

A plausible explanation for this change is uncertainty about the goals and credibility of the Federal Reserve prior to the Volcker disinflation, which was replaced by over time by increased trust in policymakers' commitment to fighting inflation. Uncertainty about the inflation target implies that people may have not only been unsure what the target was, but also what others believe the target to be, and what they believe others believe, and so on. Changes in the long-run inflation target are more important for long-run bonds because a nominal bond's real returns are eroded by sustained higher inflation. A greater commitment to fighting inflation and greater transparency may have lead to a gradual consensus about what the Fed's current stance of policy and its implicit inflation target likely was. This may have lead to the decline in the higher order wedge over time.[28]

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[27]Appendix figures 17a and 17b show the estimated paths of these variables both steadily declined from the early 1980s to the early 2000s.

[28]The importance of the credibility of the central bank's inflation target is consistent with Wright (2011). He argues changes in the conduct of monetary policy lowered inflation uncertainty and that inflation uncertainty significantly explains the five-to-ten year forward premium across his sample of countries from 1990-2009. Both the results here and in Wright's paper are consistent with the idea that lower inflation uncertainty over time has caused the premium on long-term US government debt to decline. In my model, this is a result of both the relationship between changes in the inflation target and the risk variable - that is, the direct role of the inflation target and risk - as well as (higher order) uncertainty about monetary policy arising endogenously from the traders' inference problem.
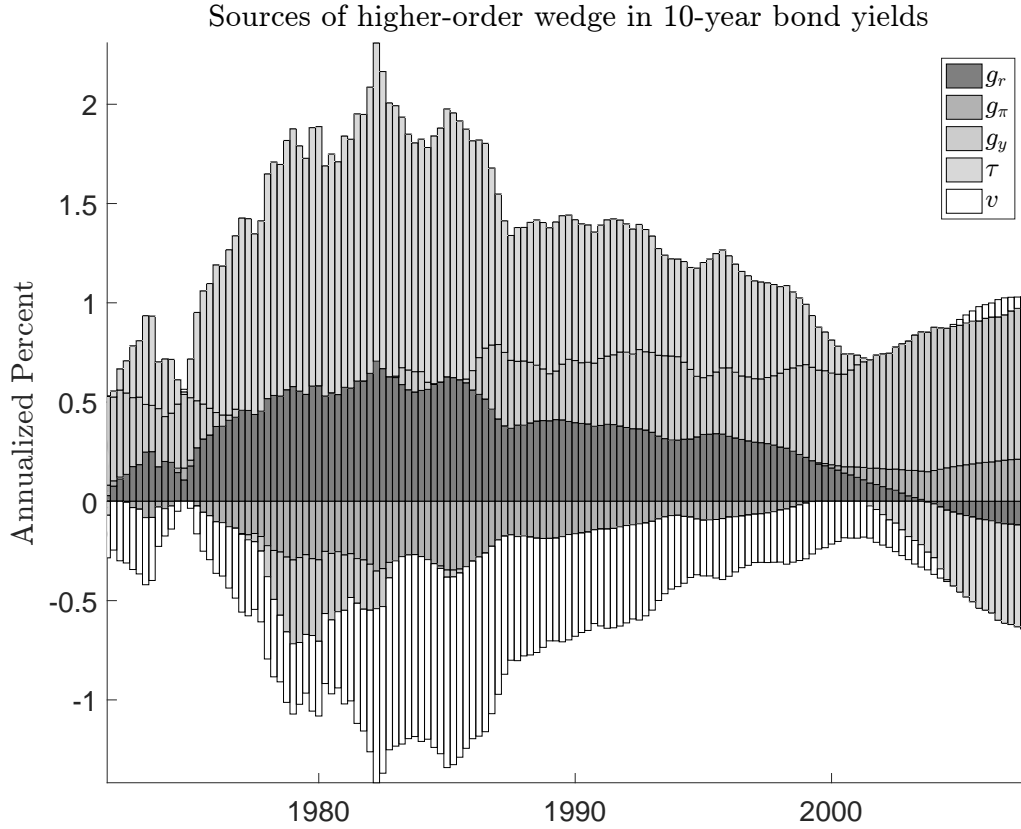
Figure 9: Decomposition of higher order wedge, ten year yields. Details of the decomposition are found in section B.6.

Examining the decomposition also reveals a degree of canceling out of higher order beliefs. This is because different risks are not perfectly correlated with each other, and agents' higher-order beliefs are constrained by the macroeconomic environment. The average and maximum contribution of higher-order expectations to yields is shown in table 5. The contribution of higher order expectations to the wedge is increasing over the maturity of the bond. This is consistent with the model intuition at the beginning of section 3. Longer-maturity bonds are a function of future expectations of future stochastic discount factors. The longer the maturity of the bond, the larger the role of higher-order beliefs in determining the price.

Table 5: Contribution of higher-order wedge to yields at the posterior mode

	1 year	2 year	3 year	4 year	5 year	10 year
Average	0.03	0.15	0.18	0.20	0.44	0.66
Maximum	0.10	0.31	0.40	0.55	0.72	0.89
	Average contributions by source:					
g_r	-0.05	-0.28	-0.41	-0.11	0.11	0.31
g_π	0.03	0.23	0.36	0.19	0.04	-0.09
g_y	0.12	0.59	0.93	0.71	0.48	0.28
τ	-0.14	-0.78	-1.18	-0.50	0.04	0.54
v	0.01	0.09	0.14	-0.08	-0.23	-0.37

Table 5 also reveals how higher-order beliefs about different risks play different roles in the wedge across different maturities. This is a result of the expected time path of higher order beliefs and how different risks are priced at different horizons. In particular, as figure 4 reveals, higher-order beliefs about the output gap tend to fall over the medium term when the inflation target rises, which (along with the estimated prices of risk) explains why during the period when τ contributes the most to the higher-order wedge for 10 year yields is also when g^y plays such a large role for 3 and 4 year bonds. For bonds of low maturity, the contributions of higher order beliefs are very small in absolute terms and essentially cancel out on average.[29]

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[29]The contribution of higher order beliefs, and their time series properties, are somewhat different here than in Barillas and Nimark (2015). They find that higher order beliefs play a larger role in general (with the peak contribution as a fraction of yields in the early 1990s) and also find a large negative role for the higher order wedge during the early 2000s. additional restrictions the structural VAR places on the risks faced by agents. Agents' beliefs about pricing factors and the role of those factors in prices are constrained by the covariances between asset prices and macroeconomic yields in the data. Their latent factor model is more flexible. A second important difference is the choice of data. Barillas and Nimark directly use SPF data on interest rate expectations to discipline belief formation, whereas I use inflation forecasts. Inflation forecasts in the SPF generally perform better than most forecasting models (Faust and Wright (2013)). This feature of the data will imply agents have better average forecasts of inflation, which may mean the choice of data generates a more conservative role for higher order beliefs. Moreover, since zero coupon yields are constructed based on estimates from prices of different kinds of outstanding Treasury debt, there may be a concern that the "model" concept of Treasury yields is different from the concept that the SPF forecasters had in mind, which might exaggerate deviations of yields from rate expectations. This could influence estimates of the higher order wedge. Furthermore, the quarterly time series for interest rate forecasts in the SPF is much shorter than the inflation forecast data and a worse response rate. Inflation forecasts are available for the whole sample period. The higher order wedge appears to play a greater influence in the Barillas and Nimark (2015) results once rate forecasts become available.

8 Conclusion

Survey evidence suggests professional forecasters have dispersed beliefs about future prices of Treasury bonds and macroeconomic variables. In this paper, I construct and estimate a structural model that reflects this feature of the world. My estimates imply that the direct role of belief dispersion is somewhat modest, but that most of the time variation in the higher order wedge is caused by policy-related factors. In particular, the wedge grew during the 1970s and early 1980s, along with the central bank's implicit inflation target, and fell over the course of the Great Moderation. This is consistent with gradual learning by agents about a new monetary policy regime and the emergence of a consensus about the conduct of monetary policy, perhaps arising from greater transparency and credibility.

I also provide new estimates of the quality of agents' private information and how much they learn from prices. I find individual private signals are quite noisy. By contrast, a great deal of agents' information about fundamentals comes from public prices, and prices are especially informative about the beliefs of others. Absent any of the public signals in the model, agents are about half as informed about macroeconomic fundamentals and know only about a fifth as much about the long-run inflation target of the central bank and financial risk. My paper is the first to measure the importance of the policy rate as a signal of fundamentals in an asset pricing setting where agents are not artificially constrained from learning from other prices.

The results add to the body of evidence that deviations from full information are an important feature of the world. Accounting for agents' inference dramatically affects the size and interpretation of term premia, even without constraints on using prices as information or assuming traders have model inconsistent beliefs. The result that asset prices appear to be an important source of macroeconomic information suggests general equilibrium macroeconomic models with dispersed information should account for learning from prices when quantifying

the importance of these frictions or when assessing normative questions. It also suggests, at least for asset prices, market consistent information is not enough for aggregate irrelevance of information frictions. This is true in two senses: Dispersed information directly affects prices and the behavior of endogenous beliefs is quite different than under full information.

There are a number of interesting and important extensions to this paper that would be worth pursuing. In this paper, I have focused on the informational content of a single type of asset - nominal government debt. Other assets may have different information implications. Extending the analysis to debt of different countries - along the lines of Wright (2011) - may also be informative about how changes in the monetary policy framework are associated with changes in the importance of higher-order beliefs. Throughout the paper I have taken advantage of the fact that yields are affine. This makes characterizing the higher order wedge and informativeness of signals straightforward. However, in the aftermath of the financial crisis, there were nonlinearities in yields introduced by the zero lower bound which may have affected prices' information content. Understanding how agents' inference changed during the zero lower bound period would be worthwhile.

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A Graphical and reduced form evidence on dispersed beliefs

In this appendix, I show the empirical distribution of one-quarter ahead forecasts of short-term interest rates, and then apply the methodology of Coibion and Gorodnichenko (2015) to interest rate forecasts as discussed in the main text.

A.1 One step ahead Treasury bill forecasts

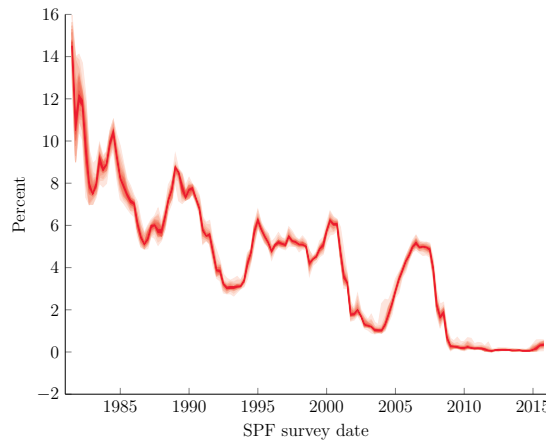**Empirical distribution of one quarter ahead forecasts of 3-month Treasury bill rates**

Figure 10: Distribution of SPF forecasts of next-quarter average rate on 3-month Treasuy bill.

A.2 A reduced form test for information rigidities

The methodology of this section follows Coibion and Gorodnichenko (2015).

For simplicity, assume Treasury bill rates follow an AR(1) process but agents observe idiosyncratic, noisy signals about the realization of that process. Innovations and signal noise are assumed to be normally distributed and mean zero:

$$
\begin{aligned}
r_t &= \rho r_{t-1} + \varepsilon_t \text{ with } \rho \in [0,1) \\
r_{it} &= r_t + e_{it}
\end{aligned}
$$

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Assuming agents are Bayesian learners, their conditional expectations can be written as:

$$E_t^i r_t = \kappa r_{it} + (1 - \kappa) E_{t-1}^i r_t$$
$$E_t^i r_{t+h} = \rho^h E_t^i r_t$$

Their expectation of the short rate is a weighted average of their current signal and their prior, where κ is the relative weight placed on the signal. As with the notation in the main model, I use $r_{t|t}^{(1)}$ to indicate the average expectation of r_t at time t.

Averaging across agents and rearranging gives the relationship between the forecast error for the *average* forecast and the revision of the average forecast at each horizon h:

$$r_{t+h} - r_{t+h|t}^{(1)} = \frac{1 - \kappa}{\kappa} \left(r_{t+h|t}^{(1)} - r_{t+h|t-1}^{(1)} \right) + \sum_{j=1}^{h} \rho^{h-j} \varepsilon_{t+h} \tag{23}$$

where the error term is the sum of rational expectations errors. If signals were perfectly informative, $\kappa = 1$, and there would be no weight on forecast revisions in this regression. To the extent agents face information frictions, $\kappa < 1$. The simple reduced-form test of information frictions in financial forecasts amounts to projecting forecast revisions on forecast errors; the null hypothesis of full information rational expectations is equivalent to testing whether the regression coefficient is 0. Finding a significant positive coefficient, on the other hand, suggests information frictions. The regression takes the form

$$\text{Average Forecast Error}_{t,h} = \beta(\text{Average Forecast Revision}_{t,h}^{t,t-1}) + \bar{\varepsilon}_t \tag{24}$$

where $\bar{\varepsilon}_t$ is the sum of rational expectations errors as before.

The results of conducting this for different forecast horizons are shown in figure 11 for 3-month Treasury bills. The results are broadly consistent with Coibion and Gorodnichenko (2015)'s findings for inflation. The estimated coefficient is positive and at least marginally significant, suggesting average forecasts for financial variables reflect dispersed information among individuals. The response for 10 year bonds (not shown) are more mixed and have a high degree of uncertainty, probably reflecting the fact that the sample of available fore-

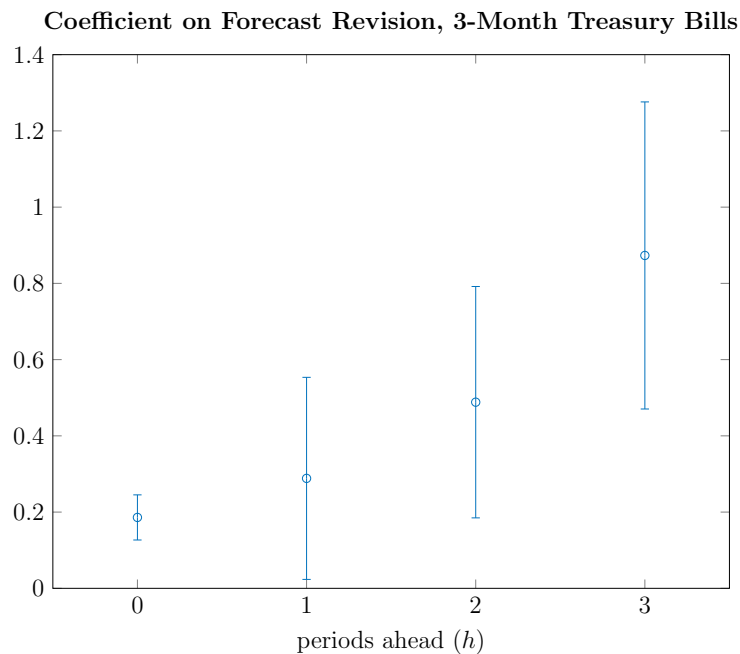**Coefficient on Forecast Revision, 3-Month Treasury Bills**

Figure 11: Caption: Coefficients from regression (24) for Treasury Bills. Bands represent 90 percent HAC confidence bands.

casts is much smaller. However, the point estimates are consistently positive and generally significant.

B Model derivations

B.1 Intuition

Beginning with

$$P_t^n = E_t^j \left[M_{t+1}^j P_{t+1}^{n-1} \right]$$

Joint lognormality implies:

$$p_t^n = E_t^j[m_{t+1}^j] + E_t^j[p_{t+1}^{n-1}] + \frac{1}{2}\mathrm{Var}(m_{t+1}^j + p_{t+1}^{n-1})$$

Iterating ahead for another agent (an arbitrary k that agent j will sell the bond to)

$$p_{t+1}^{n-1} = E_{t+1}^k[m_{t+2}] + E_{t+1}^k[p_{t+2}^{n-2}] + \frac{1}{2}\mathrm{Var}(m_{t+2}^k + p_{t+2}^{n-2})$$

Then substituting this into the price expectation term:

$$\begin{aligned}
p_t^n = {} & E_t^j[m_{t+1}^j] \\
& + E_t^j[E_{t+1}^k(m_{t+2}^k)] + E_t^j[E_{t+1}^k p_{t+1}^{n-2}] + E_t^j[E_{t+1}^k p_{t+1}^{n-2}] \\
& + \frac{1}{2}\mathrm{Var}(m_{t+1}^j + p_{t+1}^{n-1}) + \frac{1}{2}\mathrm{Var}(m_{t+2}^k + p_{t+2}^{n-2})
\end{aligned}$$

The fact that information sets are not nested means the law of iterated expectations does not apply. However, because no agent has particular information about other agents, agent j's expectations about k's expectations can be replaced by her expectation of the average expectation. Doing so, and integrating both sides over all agents implies the equation in the text.

B.2   The filtering problem

The individual agent's filtering problem, and its aggregation into the vector of average higher order expectations, follows Nimark (2007) and Barillas and Nimark (2015).

Call X_t the underlying state we want to estimate (the vector of higher order expectations, including 0th order expectations). Call $\Sigma_{t|t-1} \equiv E[(X_t - X_{t|t-1})(X_t - X_{t|t-1})']$.

Forecast step.   Given information dated time $t-1$, j's forecast of the signal is

$$z_{t|t-1}^j = \mu_Z + D X_{t|t-1} \tag{25}$$

The associated covariance matrix of signal forecasting error is

$$\begin{aligned}
\Omega_{t|t-1} &\equiv E[(z_t^j - z_{t|t-1}^j)(z_t^j - z_{t|t-1}^j)'] \\
&= D\Sigma_{t|t-1}D' + RR'
\end{aligned} \tag{26}$$

Updating step. Projection of $X_t - X_{t|t-1}$ onto $z_t^j - z_{t|t-1}^j$ and rearrangement gives that j's conditional expectation of the state given her time t information is

$$
\begin{aligned}
X_{t|t}^j &= X_{t|t-1} + \underbrace{\Sigma_{t|t-1} D' \Omega_{t|t-1}^{-1}}_{K_t} (z_t^j - z_{t|t-1}^j) \\
&= X_{t|t-1}^j + K(DX_t + R \begin{bmatrix} u_t \\ \eta_t^j \end{bmatrix} - DX_{t|t-1}^j) \\
&= \mu_X + \mathcal{F} X_{t-1|t-1}^j + K[D(\mu_X + \mathcal{F} X_{t-1} + \mathcal{C} u_t) + R \begin{bmatrix} u_t \\ \eta_t^j \end{bmatrix} - D(\mu_X + \mathcal{F} X_{t-1|t-1}^j)]
\end{aligned}
\tag{27}
$$

Deriving the aggregate law of motion. Partition R into a part associated with aggregate shocks and one associated with idiosyncratic shocks, i.e. $R \equiv \begin{bmatrix} R_u & R_\eta \end{bmatrix}'$. Integrating $X_{t|t}^j$ to obtain the vector of *average* higher order expectations "zeros out" the idiosyncratic shocks, and we're left with

$$
\begin{aligned}
X_{t|t} &= \mu_X + (\mathcal{F} - KD\mathcal{F}) X_{t-1|t-1} + K[D(\mu_X + \mathcal{F} X_{t-1} + \mathcal{C} u_t) + R_u u_t - D\mu_X + \mathcal{F} X_{t-1|t-1})] \\
&= \mu_X + (\mathcal{F} - KD\mathcal{F}) X_{t-1|t-1} + KD\mathcal{F} X_{t-1} + K(D\mathcal{C} + R_u) u_t
\end{aligned}
\tag{28}
$$

Note that these expressions have been written in terms of the steady state Kalman gain K. To find the steady state Kalman gain, we can derive the following discrete-time algebraic Riccati equation (which follows from some algebra during the updating step)

$$
\begin{aligned}
\Sigma_{t+1|t} &= E[(X_{t+1} - X_{t+1|t})(X_{t+1} - X_{t+1|t})'] \\
&= \mathcal{F}(\Sigma_{t|t-1} - \Sigma_{t|t-1} D' \Omega_{t|t-1}^{-1} D \Sigma_{t|t-1}) \mathcal{F}' + RR'
\end{aligned}
\tag{29}
$$

and iterate until convergence. The resulting steady state $\Sigma_{t+1|t}$, combined with (26), immediately implies K.

Recall that we conjectured a VAR(1) process for X_t, namely

$$
X_t \equiv \begin{bmatrix} x_t \\ X_{t|t} \end{bmatrix} = \mu_X + \mathcal{F} X_{t-1} + \mathcal{C} u_t
\tag{30}
$$

so matching coefficients we can find $\mathcal{C}, \mathcal{F}$ (recall there are d factors and we truncate at order $\bar{k}$)

$$
\begin{aligned}
\mathcal{F} &= \begin{bmatrix} F^P & \mathbf{0}_{d \times d\bar{k}} \\ \mathbf{0}_{d\bar{k} \times d} & \mathbf{0}_{d \times d\bar{k}} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{d \times d} & \mathbf{0}_{d \times d\bar{k}} \\ \mathbf{0}_{d\bar{k} \times d} & [\mathcal{F} - KD\mathcal{F}]_- \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{d \times d(\bar{k}+1)} \\ [KD\mathcal{F}]_- \end{bmatrix} \\
\mathcal{C} &= \begin{bmatrix} C & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ [K(D\mathcal{C} + R_u)]_- \end{bmatrix}
\end{aligned}
\tag{31}
$$

where $\_$ indicates truncation to ensure conformability and considering with only considering expectations up to $\bar{k}$.

B.3 Generating bond price equations

The steps here are identical to Barillas and Nimark (2015).

$$
p_t^n = A_n + B_n' X_t + \nu_t^n
$$

To arrive at this form, substitute the SDF (9) into the (log) arbitrage condition:

$$
p_t^n = \ln E \left[\exp \left\{ -r_t - \frac{1}{2} \Lambda_t^{j'} \Sigma_a \Lambda_t^j - \Lambda_t^{j'} a_{t+1}^j + p_{t+1}^{n-1} \right\} | \Omega_t^j \right]
\tag{32}
$$

Here we use the definition of a_{t+1}^j (10) to substitute p_{t+1}^{n-1} out for its expectation plus the forecast error for that particular maturity

$$
P_{t+1}^{n-1} = E \left[p_{t+1}^{n-1} | \Omega_t^j \right] + e_{n-1}' a_{t+1}^j
\tag{33}
$$

where e_n' is a horizontal selection vector with 1 in the nth element and zeros elsewhere.

Since we assumed agents knew the model equations we can write, we can write

$$
E[p_{t+1}^{n-1} | \Omega_t^j] = A_{n-1} + B_{n-1}' \underbrace{\left(\mu_X + \mathcal{F} E[X_t | \Omega_t^j] \right)}_{E[X_{t+1} | \Omega_t^j]}
\tag{34}
$$

Define an operator H that selects just the average higher order expectations from X_t^j (12), that is

$$E[X_t|\Omega_t^j] = HX_t^j$$

$$\text{where } H = \begin{bmatrix} \mathbf{0}_{d\bar{k}\times d} & I_{d\bar{k}} \\ \mathbf{0}_{d\times d} & \mathbf{0}_{d\times d\bar{k}} \end{bmatrix} \tag{35}$$

Combining these three expressions gives

$$E[p_{t+1}^n|\Omega_t^j] = A_{n-1} + B'_{n-1}\mu_X + B'_{n-1}\mathcal{F}HX_t^j \tag{36}$$

substituting this in to the no-arbitrage condition

$$p_t^n = \ln E\left[\exp\left\{-r_t - \frac{1}{2}\Lambda_t^{j\prime}\Sigma_a\Lambda_t^j - \Lambda_t^{j\prime}a_{t+1}^j + A_{n-1} + B'_{n-1}\mu_X + B'_{n-1}\mathcal{F}HX_t^j + e'_{n-1}a_{t+1}^j\right\}\Big|\Omega_t^j\right] \tag{37}$$

The inner expression consists of constants and lognormal random variables. It can be written in terms of things known to agent j at time t (so the expectation is superfluous):

$$\begin{aligned} p_t^n = \ln\exp\Big\{ &- r_t - \frac{1}{2}\Lambda_t^{j\prime}\Sigma_a\Lambda_t^j - A_{n-1} + B'_{n-1}\mu_X + B'_{n-1}\mathcal{F}HX_t^j \\ &+ \frac{1}{2}(e'_{n-1}\Sigma_a e_{n-1} + \Lambda_t^{j\prime}\Sigma_a\Lambda_t^j - 2e'_{n-1}\Sigma_a\Lambda_t^j)\Big\} \end{aligned} \tag{38}$$

where the last term is $1/2$ times the variance of $(e'_{n-1} - \Lambda_t^j)a_{t+1}^j$. Simplifying:

$$p_t^n = -r_t + A_{n-1} + B_{n-1}\mu_X + B'_{n-1}\mathcal{F}HX_t^j + \frac{1}{2}e'_{n-1}\Sigma_a e_{n-1} - e'_{n-1}\Sigma_a\Lambda_t^j \tag{39}$$

The price of the n period bond at time t is a function of constants, the current risk-free rate, and j specific terms. By no arbitrage, this expression holds for all j at all times, but, like Barillas and Nimark (2015), I focus on a hypothetical agent whose state coincides with the cross-sectional average state. Then we can substitute X_t for X_t^j in the previous expression, since $X_t \equiv \int X_t^j dj$.

Finally substitute (2) and (11) into the previous expression:

$$p_t^n = - (\delta_0 + \delta'_x x_t) + A_{n-1} + B_{n-1}\mu_X + B'_{n-1}\mathcal{F}HX_t$$
$$+ \frac{1}{2}e'_{n-1}\Sigma_a e_{n-1} - e'_{n-1}\Sigma_a \left(\Lambda_0 + \Lambda_x^j + \Lambda_\nu \int E[\nu_t|\Omega_t^j]dj\right) \tag{40}$$

Define $\delta'_X \equiv \begin{bmatrix} \delta'_x & \mathbf{0} \end{bmatrix}$ and rearrange this

$$p_t^n = - \delta_0 + A_{n-1} + B_{n-1}\mu_X + \frac{1}{2}e'_{n-1}\Sigma_a e_{n-1} - e'_{n-1}\Sigma_a\Lambda_0$$
$$- \delta'_X X_t + B'_{n-1}\mathcal{F}HX_t - e'_{n-1}\Sigma_a\Lambda_x X_t \tag{41}$$
$$- e'_{n-1}\Sigma_a\Lambda_\nu \int E[\nu_t|\Omega_t^j]dj$$

We had guessed

$$p_t^n = A_n + B'_n X_t + \nu_t^n \tag{5}$$

To arrive at the conjectured form, impose two additional restrictions. First, restrict:

$$\Lambda_\nu = -\Sigma_a^{-1} \tag{42}$$

which also reduces the number of free parameters in the model. Secondly, we can substitute to replace the remaining $e'_{n-1}\int E[\nu_t|\Omega_t^j]dj$ term via a convenient normalization. Note model consistent expectations and the conjectured bond price equation imply

$$p_t^n = E[A_n + B_n X_t + \nu_t^n|\Omega_t^j] = A_n + B_n H X_t^j + e'_{n-1}E[\nu_t|\Omega_t^j] \tag{43}$$

Setting this equal to the conjectured bond equation implies

$$A_n + B_n H X_t + e'_{n-1}\int E[v_t|\Omega_t^j]dj = \quad A_n + B_n X_t + \nu_t^n$$
$$\Rightarrow e'_{n-1}\int E[v_t|\Omega_t^j]dj = \quad B_n(I - H)X_t + \nu_t^n \tag{44}$$

Substituting these restrictions:

$$p_t^n = - \delta_0 + A_{n-1} + B_{n-1}' \mu_X + \frac{1}{2} e_{n-1}' \Sigma_a e_{n-1} - e_{n-1}' \Sigma_a \Lambda_0$$
$$- \delta_X' X_t + B_{n-1}' \mathcal{F} H X_t - e_{n-1}' \Sigma_a \Lambda_x X_t \tag{45}$$
$$+ B_n (I - H) X_t + \nu_t^n$$

Finally, write $B = \begin{bmatrix} B_2' & \cdots & B_{\tilde{n}}' \end{bmatrix}$ and note that $B_n = e_{n-1} B$. Normalizing prices of risk:

$$\Lambda_x = \widehat{\Lambda}_x + B(I - H) \tag{46}$$

and then

$$p_t^n = - \delta_0 + A_{n-1} + B_{n-1}' \mu_X + \frac{1}{2} e_{n-1}' \Sigma_a e_{n-1} - e_{n-1}' \Sigma_a \Lambda_0$$
$$- \delta_X' X_t + B_{n-1}' \mathcal{F} H X_t - e_{n-1}' \Sigma_a \widehat{\Lambda}_x X_t \tag{47}$$
$$+ \nu_t^n$$

This implies the recursive forms for the bond price equations:

$$A_{n+1} = -\delta_0 + A_n + B_n \mu_X + \frac{1}{2} e_n' \Sigma_a e_n - e_n' \Sigma_a \Lambda_0 \tag{48}$$
$$B_{n+1}' = -\delta_X + B_n' \mathcal{F} H - e_{n-1}' \Sigma_a \widehat{\Lambda}_x \tag{49}$$
$$\text{with}$$
$$A_1 = -\delta_0 \tag{50}$$
$$B_1 = -\delta_X' \tag{51}$$

which implies $p_t^1 = -\delta_0 + [\delta_x, \mathbf{0}] X_t = -r_t$.

B.4   Macroeconomic structure

$$P_0 = \begin{bmatrix} 1 & -(1 - \phi_r)\phi_\pi & -(1 - \phi_r)\phi_y & 0 & -(1 - \phi_r)\phi_v \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{52}$$

$$\mu_x = \begin{bmatrix} (1 - \phi_r)g^r - (1 - \phi_r)g_y \\ -\rho_{\pi r}g_r - \rho_{\pi y}g^y \\ g_y - \rho_{yr}g^r - \rho_{yy}g^y \\ (1 - \rho_{\tau\tau})\tau \\ 0 \end{bmatrix} \tag{53}$$

$$P_1 = \begin{bmatrix} \phi_r & 0 & 0 & 0 & 0 \\ \rho_{\pi r} & \rho_{\pi\pi} & \rho_{\pi y} & 0 & \rho_{\pi v} \\ \rho_{yr} & \rho_{y\pi} & \rho_{yy} & 0 & \rho_{yv} \\ 0 & 0 & 0 & \rho_{\tau\tau} & 0 \\ 0 & 0 & 0 & 0 & \rho_{vv} \end{bmatrix} \tag{54}$$

$$\Sigma_0 = \begin{bmatrix} \sigma_r & 0 & 0 & 0 & 0 \\ 0 & \sigma_\pi & 0 & \sigma_{\pi\tau}\sigma_\tau & 0 \\ 0 & \sigma_{y\pi}\sigma_\pi & \sigma_{y\pi}\sigma_\pi & \sigma_{y\tau}\sigma_\tau & 0 \\ 0 & 0 & 0 & \sigma_\tau & 0 \\ \sigma_{vr} & \sigma_{v\pi} & \sigma_{vy} & \sigma_{v\tau} & \sigma_v \end{bmatrix} \tag{55}$$

$$r_t = \delta_0 + \delta_x' x_t \tag{2}$$

with

$$\delta_0 = \mathbf{0}_{5\times 1} \tag{56}$$

$$\delta_x' = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{57}$$

and the matrices governing the evolution of fundamentals (3) as

$$\begin{aligned} \mu^P &= P_0^{-1}\mu_0 \\ F^P &= P_0^{-1}P_1 \\ C &= P_0^{-1}\Sigma_0 \end{aligned} \tag{58}$$

B.5   Restrictions on Prices of Risk

Recall expressions for the stochastic discount factor and prices of risk:

$$m_{t+1}^j = -r_t - \frac{1}{2}\Lambda_t^{j\prime}\Sigma_a\Lambda_t^j - \Lambda_t^{j\prime}a_{t+1}^j \tag{9}$$

$$\Lambda_t^j = \Lambda_0 + \Lambda_x X_t^j + \Lambda_\nu E[\nu_t|\Omega_t^j] \tag{11}$$

To impose the Ireland (2015) restriction, I set:

$$\lambda_0 = \begin{bmatrix} \lambda^r & \lambda^\pi & \lambda^y & \lambda^\tau & 0 \end{bmatrix}' \tag{59}$$

$$\lambda_x = \begin{bmatrix} 0 & 0 & 0 & 0 & \lambda_x^r \\ 0 & 0 & 0 & 0 & \lambda_x^\pi \\ 0 & 0 & 0 & 0 & \lambda_x^y \\ 0 & 0 & 0 & 0 & \lambda_x^\tau \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{60}$$

To additionally impose the Barillas and Nimark (2015) restriction, recall that the vector of bond price innovations a_{t+1}^j is a linear combination of forecasting error in the factors X_{t+1} and maturity-specific price shocks ν_{t+1}.

$$a_{t+1}^j = \Psi \begin{bmatrix} X_{t+1} - E\left[X_{t+1}|\Omega_t^j\right] \\ \nu_{t+1} \end{bmatrix} \tag{61}$$

To see this, write j's one-period ahead bond pricing error for a particular maturity as

$$\begin{aligned} a_{t+1}^{n,j} &= p_{t+1}^{n-1} - p_{t+1|t}^j \\ &= B_{n-1}'(X_{t+1} - E_t^j X_t) + \nu_{t+1}^{n-1} \end{aligned} \tag{62}$$

So stacking these errors in a vector a_{t+1}^j gives

$$a_{t+1}^j = \underbrace{\begin{bmatrix} B_1' & \\ \vdots & I_{\bar{n}-1} \\ B_{\bar{n}-1}' & \end{bmatrix}}_{\Psi} \begin{bmatrix} X_{t+1} - E\left[X_{t+1}|\Omega_t^j\right] \\ \nu_{t+1} \end{bmatrix} \tag{63}$$

Left multiplying by $\Lambda_{t+1}^{j\prime}$:

$$\Lambda_{t+1}^{j\prime} a_{t+1}^j = \Lambda_{t+1}^{j\prime} \Psi \begin{bmatrix} X_{t+1} - E\left[X_{t+1}|\Omega_t^j\right] \\ \nu_{t+1} \end{bmatrix} \tag{64}$$

We want to restrict this so that

$$\Lambda_{t+1}^{j\prime} a_{t+1}^j = \left(\begin{bmatrix} \lambda_0 \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \lambda_x & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \right)' \begin{bmatrix} X_{t+1} - E\left[X_{t+1}|\Omega_t^j\right] \\ \nu_{t+1} \end{bmatrix} \tag{65}$$

If we removed dispersed information or maturity-specific shocks, this restriction would imply only fundamentals matter for bond prices, given the restrictions in (59) and (60). When maturity specific shocks are equal to zero, these additional restrictions must hold:

$$\begin{aligned} \begin{bmatrix} \lambda_0 \\ \mathbf{0} \end{bmatrix}' \begin{bmatrix} X_{t+1} - E\left[X_{t+1}|\Omega_t^j\right] \\ \nu_{t+1} \end{bmatrix} &= \Lambda_0' a_{t+1}^j \\ \left(\begin{bmatrix} \lambda_x & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} X_t^j \right)' \begin{bmatrix} X_{t+1} - E\left[X_{t+1}|\Omega_t^j\right] \\ \nu_{t+1} \end{bmatrix} &= \widehat{\Lambda}_x' a_{t+1}^j \end{aligned} \tag{66}$$

where $\widehat{\Lambda}_x$ is a normalization (see appendix B.3). This can be achieved by setting

$$\begin{aligned} \Phi &= \Psi(\Psi'\Psi)^{-1} \\ \Lambda_0 &= \Phi \begin{bmatrix} \lambda_0 \\ \mathbf{0} \end{bmatrix} \\ \widehat{\Lambda}_x &= \Phi \begin{bmatrix} \lambda_x & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \\ \Psi &= \begin{bmatrix} B_1' & \\ \vdots & I_{\bar{n}-1} \\ B_{\bar{n}-1}' & \end{bmatrix} \end{aligned} \tag{67}$$

These restrictions are the same as those imposed in Barillas and Nimark (2015).

B.6   Bond price decompositions

Given the expression for prices and a model for inference, we can characterize what portion of bond yields are driven by higher-order beliefs - that is, the portion of yields driven directly by dispersed information. Common knowledge of rationality and fact that X_t has a Markov structure implies (1) bond prices are pinned down by the *current* state and thus

agents' forecasts of *future* states determine their forecasts of future bond prices, and (2) all information about future X_t is summarized in today's state (Barillas and Nimark (2015)). Hence, two agents who agree about X_t agree about X_{t+1}, X_{t+2}, etc, and thus agree about price forecasts. Intuitively, the difference between actual prices and the price that would obtain if all agents counterfactually held the same beliefs is the direct contribution of dispersed information to the bond price.[30] Like Barillas and Nimark (2015), I use the wedge between the counterfactual price with common beliefs and actual prices to quantify the extent to which dispersed information about particular factors affects bond yields. Moreover, because priced risks have a macroeconomic interpretation, the wedge can be decomposed in order to understand whether disagreement about particular macroeconomic conditions are important for determining yields at different maturities.

Define a matrix operator $\bar{H}$ that replaces all higher order expectations with first order expectations, that is:

$$
\begin{bmatrix} x_t \\ x_t^{(1)} \\ \vdots \\ x_t^{(1)} \end{bmatrix} = \bar{H} \begin{bmatrix} x_t \\ x_t^{(1)} \\ \vdots \\ x_t^{(\bar{k})} \end{bmatrix}
\tag{68}
$$

The price that would obtain if all higher order expectations coincided with the first order expectation - the "counterfactual consensus price" - is

$$
\bar{p}_t^n = A_n + B_n' \bar{H} X_t + \nu_t^n
\tag{69}
$$

We can use this to decompose prices into the component that depends on averag (first order) expectations and the component that depends on dispersion of information and the resulting divergence of expectations about expectations. The wedge can be written as:

---

[30] Allen et al. (2006) show in a similar setting how prices of long-lived assets will not generally reflect average expectations when there is private information. Barillas and Nimark (2015) refer to the difference between actual prices and the counterfactual consensus price as the "speculative component"; Bacchetta and Van Wincoop (2006) refer to it as the "higher order wedge." The preferred interpretation of Bacchetta and Van Wincoop is that it is the present value of deviations of higher-order beliefs from first-order beliefs.

$$p_t^n - \bar{p}_t^n = B_n' X_t - B_n' \bar{H} X_t = B_n'(I - \bar{H})X_t \tag{70}$$

The counterfactual consensus price, which contains only the effect of average expectations in yields, can be decomposed into short rate expectations and "classical" risk premia - that is, the part of yields that depends on first-order average beliefs net of average rate expectations.

$$p_t^n = A_n^{\text{prem}} + B_n^{\text{prem}\prime} X_t + A_n^{\text{rate}} + B_n^{\text{rate}\prime} X_t + \underbrace{B_n'(I - \bar{H})X_t}_{\text{higher order wedge}} + \nu_t^n \tag{71}$$

Where $A_n^{\text{prem}} = A_n - A_n^{\text{rate}}, B_n^{\text{prem}\prime} = B_n' \bar{H} - B_n^{\text{rate}\prime}$. To make this decomposition operative, we need the model-implied future expected short rates. For the hypothetical average agent,

$$E_{t|t} r_{t+1} = -\delta_0 - \delta_X H X_{t+1|t} = -\delta_0 - \delta_X(\mu_X + \mathcal{F}H X_t)$$

and so on for further ahead future short rates:

$$A_n^{\text{rate}} = - n(\delta_0 + \delta_X \mu_X) - \delta_X \sum_{s=0}^{n-1} \mathcal{F}^s \mu_X$$

$$B_n^{\text{rate}\prime} = - \delta_X \sum_{s=0}^{n-1} \mathcal{F}^s H \tag{72}$$

The decomposition of the wedge is a straightforward selection of different elements. For example, the portion of the higher-order wedge attributable to higher-order beliefs about the long-run inflation target τ_t is

$$B_n'(I - \bar{H})X_t^\tau \equiv$$
$$B_n'(I - \bar{H}) \cdot \text{diag} \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 \end{bmatrix} X_t \tag{73}$$

Note that this depends on both the level of the (higher order) expectations (i.e., $\tau_t^{(2)}, \tau_t^{(3)}$) and so on), and how that level translates into compensation for risk (from B_n').

C   Fixed point procedure

0. Given a set of parameters, we construct H using (35), δ_0 δ_x, μ^P, F^P, C, λ_0 and λ_x using (56)-(60). We need an initial guess of B (typically starting with the full information B). This implies an initial A_n using (13), and thus D for the agents' filtering problem. We must also guess $\mathcal{C}, \mathcal{F}$, typically at the full information solution.

1. The Kalman filtering problem implies steady state $\Sigma_{t+1|t}$ using (29).[31] This implies steady state $\Omega_{t+1|t}$ and K. Construct $\mathcal{F}, \mathcal{C}$ from (31).

2. We have

$$\Sigma^a = \Psi \begin{bmatrix} \Sigma_{t+1|t} & \mathbf{0} \\ \mathbf{0} & \Sigma_\nu \end{bmatrix} \Psi' \tag{74}$$

where Σ_ν is the covariance matrix of maturity shocks, a diagonal matrix where the nonzero elements are of the form

$$\sqrt{\mathrm{Var}(e_t^n)} = n\sigma_\nu \tag{75}$$

(this implies the variance of maturity shocks is constant across yields, which reduces the number of free parameters). Recall we had assumed $\Lambda_\nu = -\Sigma_a^{-1}$.

3. Update our guess of B using (14) and check for convergence. If $B, \mathcal{C}, \mathcal{F}$ have converged, stop. Else, go to step 1.

---

[31]In practice, the bulk of time spent on the solution is in this step. Since no closed form exists for the Kalman gain in a general multivariate setting, I must numerically find the Kalman gain by solving the discrete-time algebraic Riccati equation. In this particular setting, the fastest way to solve the equation seems to be through iteration until convergence, with an additional step to ensure that the matrix is symmetric. The latter step is necessary to avoid numerical problems due to round-off which is common in large-dimension Kalman filtering problems.

D   Econometric matrices

The model-consistent notion of dispersion of signals around the average comes from agents'
Kalman filtering equations. Any dispersion in belief must come from idiosyncratic signals.
The idiosyncratic error covariance matrix is the solution to the following Riccati equation:

$$
\begin{aligned}
\Sigma_j &= E[(X_t^j - X_t^{(1)})(X_t^j - X_t^{(1)})'] \\
&= (\mathcal{F} - KD\mathcal{F})\Sigma_j(\mathcal{F} - KD\mathcal{F})' + KR_\eta R_\eta' K'
\end{aligned}
\tag{76}
$$

Hence the cross-sectional variance in average forecasts is just the appropriate element of Σ_j:

$$
Var(\pi_{t|t}^j) = \underbrace{[0, 1, 0, 1, 0, \mathbf{0}_{1 \times d*(\bar{k})}]}_{\equiv e^\pi} \Sigma_j e^{\pi\prime}
\tag{77}
$$

The non-constant parts of the econometric matrices in (22) are:

$$
\bar{D}_t =
\begin{bmatrix}
I_4 & \mathbf{0}_{4\times 2} & \mathbf{0}_{4\times\bar{k}} \\
& -\frac{1}{4}B_4' & \\
& -\frac{1}{8}B_8' & \\
& -\frac{1}{12}B_{12}' & \\
& -\frac{1}{16}B_{16}' & \\
& -\frac{1}{20}B_{20}' & \\
& -\frac{1}{40}B_{40}' & \\
& e^\pi\mathcal{F} \times I_{m_t^1} & \\
& e^\pi \cdot \mathcal{F}^4 \times I_{m_t^4 s} &
\end{bmatrix}
\tag{78}
$$

$$
\bar{R}_t =
\begin{bmatrix}
\mathbf{0}_{3\times d+\bar{n}-1+m_t^1} \\
\sigma_\nu
\begin{bmatrix}
& e_3 \\
& e_7 \\
& e_{11} \\
& e_{15} \\
\mathbf{0}_{6\times d} & e_{19} \\
& e_{39}
\end{bmatrix} \\
\sqrt{e^\pi\mathcal{F}\Sigma^j\mathcal{F}'e^{\pi\prime}} \times I_{m_t^1} \\
\sqrt{e^\pi\mathcal{F}^4\Sigma^j(\mathcal{F}^4)'e^{\pi\prime}} \times I_{m_t^4}
\end{bmatrix}
\tag{79}
$$

For the full information model, the equations are the same. However, instead of the σ_ν

terms, the observed bond yields are assumed to be observed with yield-specific error.[32], and the cross-sectional estimation error terms for the forecasts are replaced with horizon-specific error terms $\widetilde{\sigma}_\pi^h, h = 1, 4$.

E   Dispersed-information parameter results

---

[32]A common practice to avoid a stochastic singularity problem, used by Ireland (2015) among others, is to assume that only certain yields are observed with error. However, as Piazzesi (2009) points out, which set of yields to treat as viewed with error is essentially arbitrary, and assuming all of them are viewed with error does not pose any computational difficulty in this setting.

	Mode	Mean	Median	5%	95%	Std.
ϕ_r	0.5357	0.5339	0.5355	0.5324	0.5370	0.0224
ϕ_π	0.1835	0.1851	0.1869	0.1768	0.1908	0.0090
ϕ_y	0.1016	0.1010	0.1010	0.0928	0.1101	0.0066
ϕ_v	0.0349	0.0343	0.0348	0.0305	0.0365	0.0024
σ_r	0.0029	0.0033	0.0032	0.0029	0.0038	0.0003
ρ_{yr}	-0.9329	-0.9954	-0.9994	-1.0440	-0.9312	0.0539
$\rho_{y\pi}$	-0.3871	-0.4088	-0.4134	-0.4361	-0.3730	0.0254
ρ_{yy}	0.9164	0.8988	0.9031	0.8794	0.9136	0.0389
ρ_{yv}	-0.0007	-0.0008	-0.0006	-0.0022	-0.0000	0.0007
$\sigma_{y\pi}$	0.3329	0.2873	0.2711	0.2443	0.3355	0.0362
$\sigma_{y\tau}$	2.6888	2.6788	2.6878	2.6491	2.7163	0.1140
σ_y	0.0062	0.0068	0.0068	0.0063	0.0071	0.0004
σ_τ	0.0008	0.0009	0.0009	0.0008	0.0010	0.0001
$\rho_{\pi r}$	0.9229	0.8949	0.8882	0.8650	0.9323	0.0453
$\rho_{\pi\pi}$	0.4331	0.4368	0.4372	0.4223	0.4564	0.0208
$\rho_{\pi y}$	-0.1998	-0.1870	-0.1872	-0.2027	-0.1724	0.0117
$\rho_{\pi v}$	-0.1139	-0.1105	-0.1114	-0.1143	-0.1062	0.0055
$\sigma_{\pi\tau}$	-0.1380	-0.1483	-0.1452	-0.1993	-0.1123	0.0309
σ_π	0.0031	0.0034	0.0033	0.0031	0.0037	0.0002
ρ_{vv}	0.8633	0.8600	0.8613	0.8589	0.8640	0.0360
σ_{vr}	9.7831	9.8318	9.8834	9.7764	9.8987	0.4138
$\sigma_{v\pi}$	2.1485	2.1574	2.1672	2.1123	2.1919	0.0932
σ_{vy}	-2.1067	-2.1144	-2.1111	-2.1939	-2.0638	0.0972
$\sigma_{v\tau}$	0.8502	0.8581	0.8577	0.8215	0.9125	0.0466
λ_r	1.3216	1.0637	1.1132	0.7289	1.3235	0.1872
λ_π	-4.6156	-4.6789	-4.6655	-4.9244	-4.4789	0.2289
λ_y	0.0223	-0.5522	-0.5930	-1.0610	-0.0260	0.3227
λ_τ	-0.1725	-0.2202	-0.1318	-0.7908	0.1247	0.2885
λ_r^x	18.0439	18.3595	18.5122	17.9321	18.8519	0.8392
λ_π^x	-76.2162	-76.1999	-76.2163	-76.6236	-76.1171	3.1893
λ_y^x	-17.4621	-17.4327	-17.4515	-17.9745	-17.0402	0.7816
λ_τ^x	0.0126	0.1011	0.0987	-0.0640	0.3192	0.1232
σ_ν	0.0025	0.0032	0.0031	0.0025	0.0041	0.0005
$\widetilde{\sigma}_\pi$	0.3418	0.4817	0.5082	0.3303	0.6348	0.1106
$\widetilde{\sigma}_y$	0.7511	0.6247	0.6539	0.4601	0.7715	0.1073
$\widetilde{\sigma}_\tau$	0.5445	0.5860	0.5825	0.4727	0.7032	0.0849
$\widetilde{\sigma}_v$	0.4116	0.4153	0.4175	0.3039	0.5054	0.0609

Table 6: Parameter estimates for dispersed information model

F   Information-theoretic concepts

In the discussion of the share of information coming from private signals in section 6, I refer to a number of concepts from information theory, which I detail here without proof; more details are found in Veldkamp (2011) and Cover and Thomas (2006). As described in section 6, I characterize the extent to which variables are informative using the notion of entropy - the amount of information required to describe a random variable (Cover and Thomas (2006)). Entropy is typically expressed in terms of "bits," i.e., in terms of log base 2 units, which is convenient because the entropy of a fair coin toss is 1 bit. Intuitively, the entropy of a random variable in bits is the number of $0 - 1$ binary signals required on average to describe its realization.

The entropy of a normally distributed variable.   If x is a normally distributed variable with variance σ^2, its entropy is $\frac{1}{2}\log_2(2\pi e\sigma^2)$ (Cover and Thomas, 2006, Chapter 8).

Conditional entropy.   Conditional entropy $H(x|y)$ is a measure of how much information it takes to describe x given that y is known (Veldkamp, 2011, Chapter 3.2). It is defined as the joint entropy of x, y minus the entropy of y, that is $H(x|y) = H(x,y) - H(y)$. The calculation of the conditional entropy of a normal variable is analogous to the unconditional case, replacing the variance with the conditional variance (Veldkamp (2011)).

Mutual information.   The mutual information of two variables x and y, $I(x;y)$ is the measure of the amount of information one contains about the other. It can be calculated in terms of entropies ((Cover and Thomas, 2006, Theorem 2.4.1))):

$$I(x,y) = H(x) - H(x|y) = H(y) - H(y|x)$$

Measure of signal use. Similar to Melosi (2017, 2014), I use the "share" of mutual information as my characterization of how much information about a variable comes from (a particular subset) of signals ω_{red}. In particular, the "share" of information about a variable x used by an agent is:

$$Share_x = I(x; \omega_{red})/I(x; \omega_{full)}$$

where ω_{red} is the reduced set of signals (for example, only private signals without the use of bond prices) and ω_{full} is the complete set of signals detailed in section 3.3.

In practice, conditional variances needed to calculate mutual information are taken as particular entries from agents' state nowcasting error matrix ($\Sigma_{t|t}$) (see appendix B.2). The conditional variance of the subset of signals is calculated by solving the filtering problem of the agent assuming they have a "counterfactual" subset of signals (just as described in appendix B.2, using $A, B, \mathcal{F}, \mathcal{C}$ from the actual model solution.

Note that this share is bounded between 0 and 1 because, on average, conditioning must reduce entropy (Cover and Thomas, 2006, Theorem 2.6.5).

G   Priors

Table 7: Prior distribution of model parameters

Parameter	Prior distribution	Prior mean	Prior s.d.	Model
ϕ_r	Beta	0.5000	0.0500	
ϕ_π	Gamma	0.5000	0.3000	
ϕ_y	Gamma	0.5000	0.3000	
ϕ_v	Trunc. Normal	0.0000	0.5000	
σ_r	Inverse Gamma	0.0050	0.2000	
ρ_{yr}	Normal	-1.0000	0.5000	
$\rho_{y\pi}$	Normal	0.0000	0.5000	
ρ_{yy}	Inverse Gamma	0.9000	0.2000	
ρ_{yv}	Trunc. Normal	0.0000	2.0000	
$\sigma_{y\pi}$	Normal	0.0000	1.0000	
$\sigma_{y\tau}$	Normal	0.0000	1.0000	
σ_y	Inverse Gamma	0.1000	3.0000	
σ_τ	Inverse Gamma	0.0050	0.3000	
$\rho_{\pi r}$	Normal	0.0000	2.0000	
$\rho_{\pi\pi}$	Inverse Gamma	0.9000	0.2000	
$\rho_{\pi y}$	Normal	0.0000	0.5000	
$\rho_{\pi v}$	Normal	0.0000	2.0000	
$\sigma_{\pi\tau}$	Normal	0.0000	3.0000	
σ_π	Inverse Gamma	0.0050	0.3000	
ρ_{vv}	Beta	0.8000	0.1000	
σ_{vr}	Normal	0.0000	3.0000	
$\sigma_{v\pi}$	Normal	0.0000	3.0000	
σ_{vy}	Normal	0.0000	3.0000	
$\sigma_{v\tau}$	Normal	0.0000	3.0000	
λ_r	Uniform(-100,100)			
λ_π	Uniform(-100,100)			
λ_y	Uniform(-100,100)			
λ_τ	Uniform(-100,100)			
λ_r^x	Uniform(-100,100)			
λ_π^x	Uniform(-100,-0.001)			
λ_y^x	Uniform(-100,100)			
λ_τ^x	Uniform(-100,100)			
σ_ν	Uniform(0.001,0.02)			DI
$\widetilde{\sigma_\pi}$	Uniform(0.001,100)			DI
$\widetilde{\sigma_y}$	Uniform(0.001,100)			DI
$\widetilde{\sigma_\tau}$	Uniform(0.001,100)			DI
$\widetilde{\sigma_v}$	Uniform(0.001,100)			DI
$\widetilde{\sigma}_4$	Inverse Gamma	1.0000	3.0000	FI
$\widetilde{\sigma}_8$	Inverse Gamma	1.0000	3.0000	FI
$\widetilde{\sigma}_{12}$	Inverse Gamma	1.0000	3.0000	FI
$\widetilde{\sigma}_{16}$	Inverse Gamma	1.0000	3.0000	FI
$\widetilde{\sigma}_{20}$	Inverse Gamma	1.0000	3.0000	FI
$\widetilde{\sigma}_{40}$	Inverse Gamma	1.0000	3.0000	FI
$\widetilde{\sigma}_\pi^1$	Inverse Gamma	1.0000	3.0000	FI
$\widetilde{\sigma}_\pi^4$	Inverse Gamma	1.0000	3.0000	FI

H   Additional Results, dispersed information model

H.1   Impulse Responses
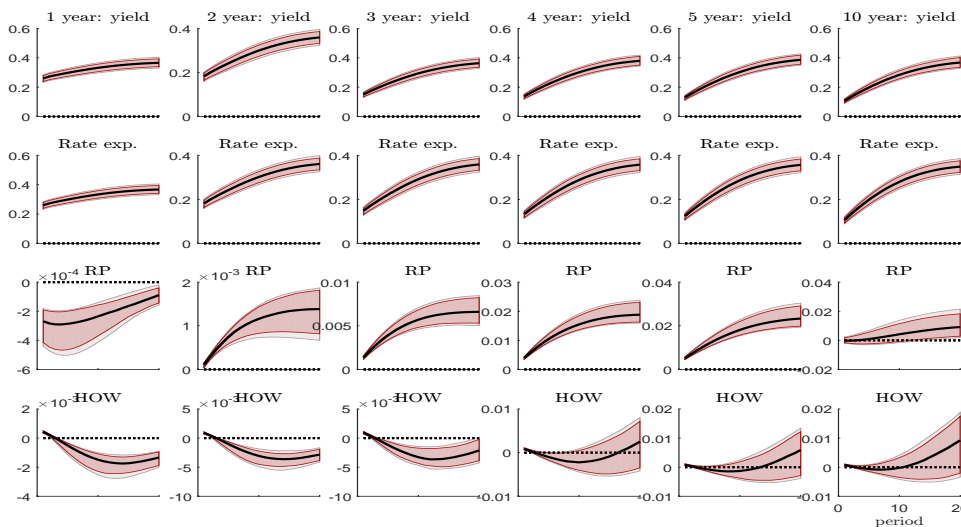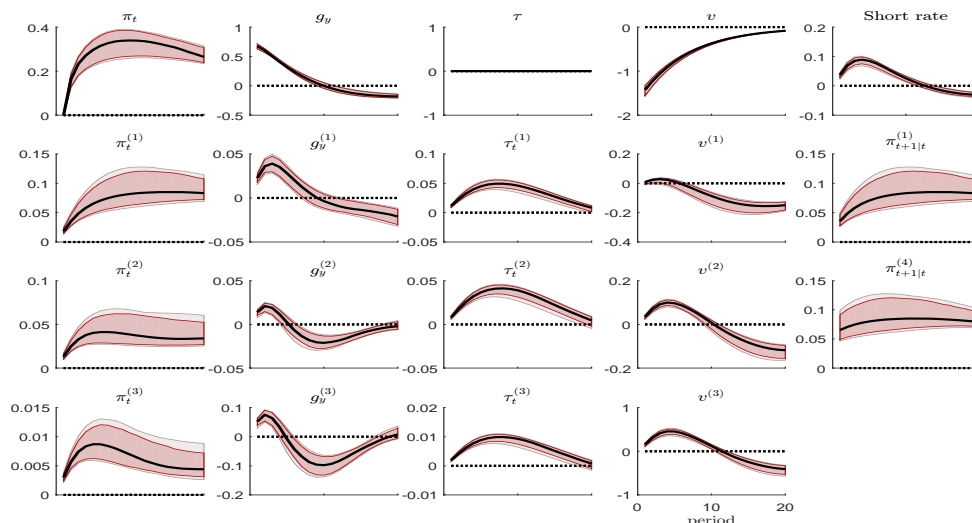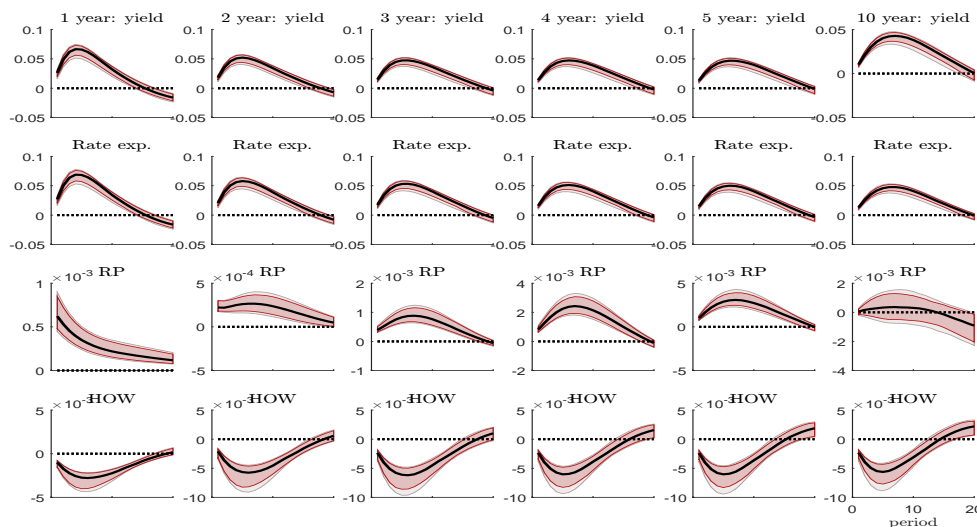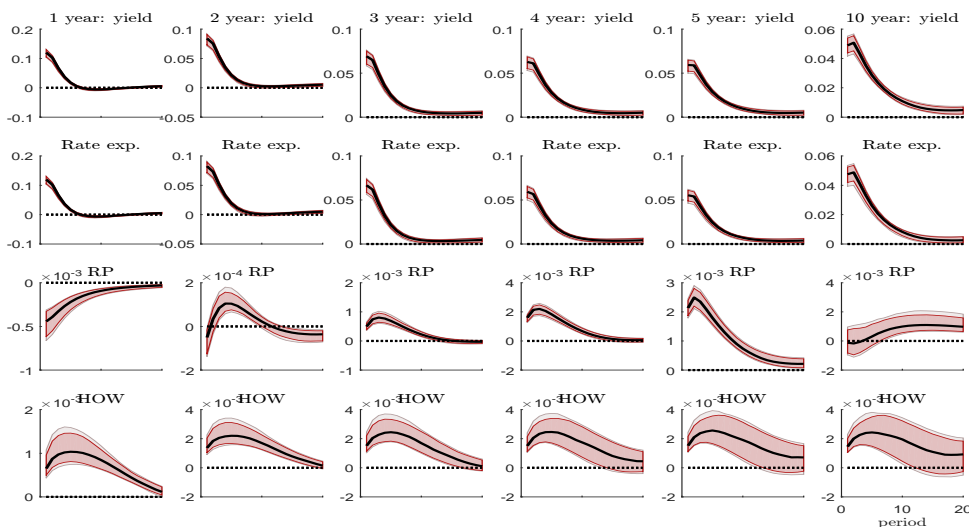
Rate shock: Macroeconomic Responses

(a) Response of non-financial variables to monetary policy rule shock, dispersed information. Black line represents the median response. Dark red indicates 80% posterior credible set. Lighter bands indicate the 90% posterior credible set. Inflation and interest rates are shown in annualized percentage points. The output gap is shown in percentage points. v is scaled by 100.
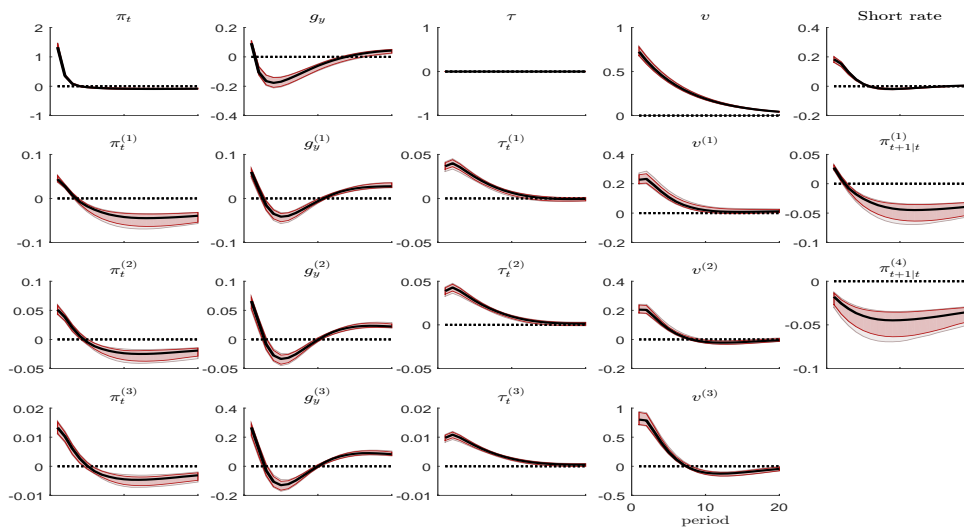
Rate shock: Yield Responses

(b) Response of financial variables to monetary policy rule shock, dispersed information. Black line represents the median response. Dark red indicates 80% posterior credible set. Lighter bands indicate the 90% posterior credible set. Inflation and interest rates are shown in annualized percentage points. The output gap is shown in percentage points. v is scaled by 100.
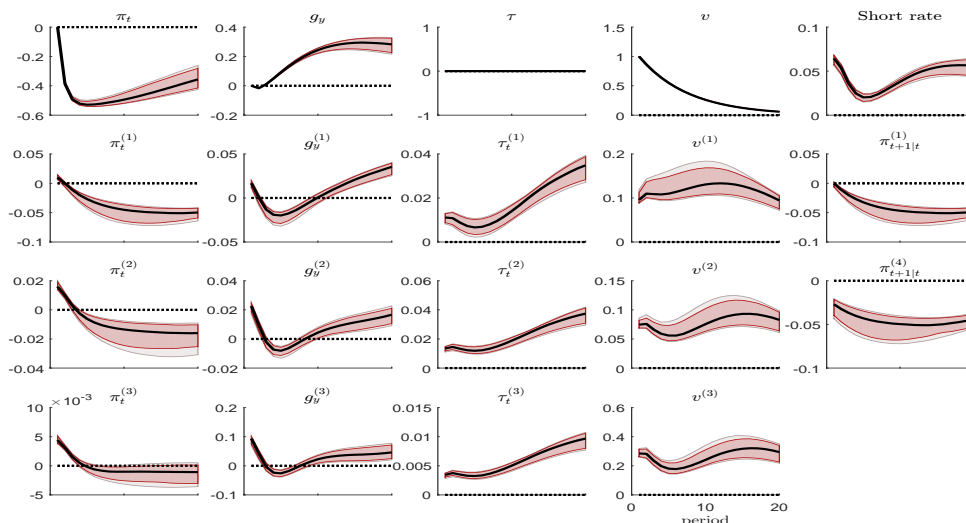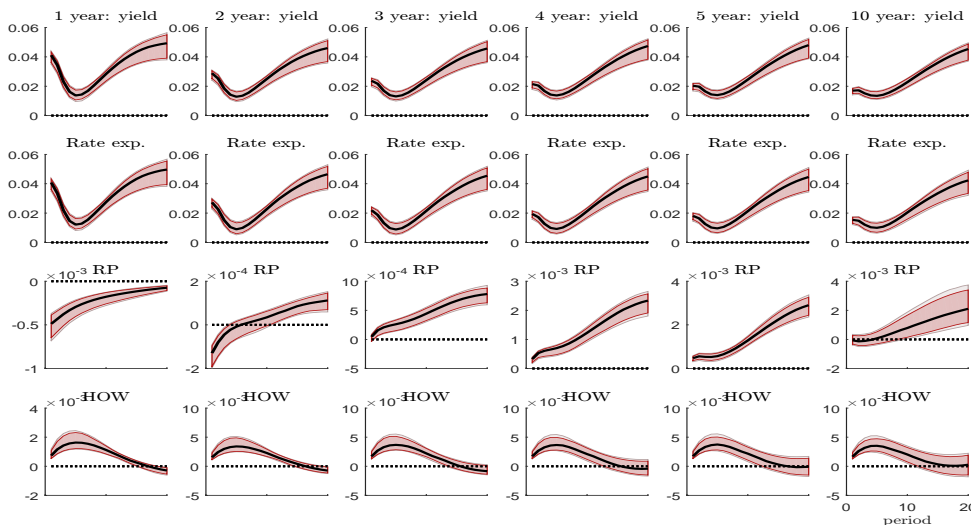
Target shock: Macroeconomic Responses

(a) Response of non-financial variables to inflation target shock, dispersed information. Black line represents the median response. Dark red indicates 80% posterior credible set. Lighter bands indicate the 90% posterior credible set. Inflation and interest rates are shown in annualized percentage points. The output gap is shown in percentage points. v is scaled by 100.

Target shock: Yield Responses

(b) Response of financial variables to inflation target shock, dispersed information. Black line represents the median response. Dark red indicates 80% posterior credible set. Lighter bands indicate the 90% posterior credible set. Inflation and interest rates are shown in annualized percentage points. The output gap is shown in percentage points. v is scaled by 100.

Output gap shock: Macroeconomic Responses

(a) Response of non-financial variables to output gap shock, dispersed information. Black line represents the median response. Dark red indicates 80% posterior credible set. Lighter bands indicate the 90% posterior credible set. Inflation and interest rates are shown in annualized percentage points. The output gap is shown in percentage points. v is scaled by 100.

Output gap shock: Yield Responses

(b) Response of financial variables to output gap shock, dispersed information. Black line represents the median response. Dark red indicates 80% posterior credible set. Lighter bands indicate the 90% posterior credible set. Interest rates are shown in annualized percentage points.

Inflation shock: Yield Responses

(a) Response of non-financial variables to inflation shock, dispersed information. Black line repre-sents the median response. Dark red indicates 80% posterior credible set. Lighter bands indicate the 90% posterior credible set. Inflation and interest rates are shown in annualized percentage points. The output gap is shown in percentage points. v is scaled by 100.

Inflation shock: Macroeconomic Responses

(b) Response of financial variables to inflation shock, dispersed information. Black line represents the median response. Dark red indicates 80% posterior credible set. Lighter bands indicate the 90% posterior credible set. Interest rates are shown in annualized percentage points.

Risk shock: Macroeconomic Responses

(a) Response of non-financial variables to risk shock, dispersed information. Black line represents the median response. Dark red indicates 80% posterior credible set. Lighter bands indicate the 90% posterior credible set. Inflation and interest rates are shown in annualized percentage points. The output gap is shown in percentage points. v is scaled by 100.

Risk shock: Yield Responses

(b) Response of financial variables to risk shock, dispersed information. Black line represents the median response. Dark red indicates 80% posterior credible set. Lighter bands indicate the 90% posterior credible set. Interest rates are shown in annualized percentage points.

H.2   State Estimates and Yield Decompositions
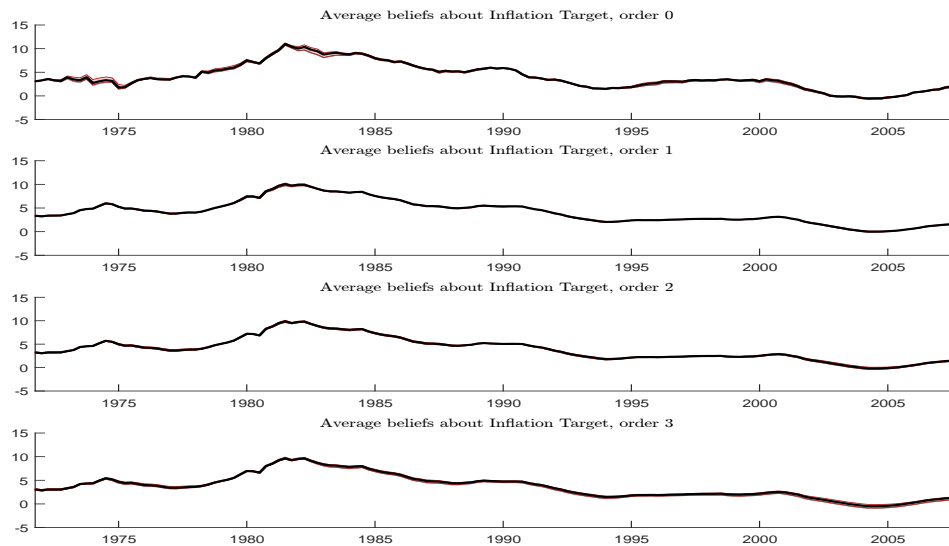
(a) Filtered estimate of inflation target and first three orders of expectation an annualized percent, dispersed information model.
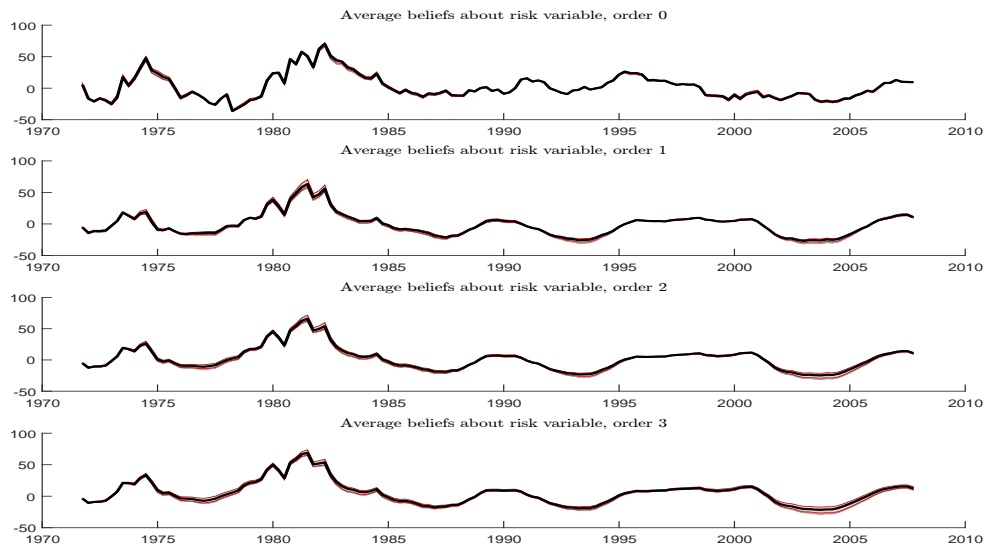
(b) Filtered estimate of risk variable and first three orders of expectation, dispersed information model.
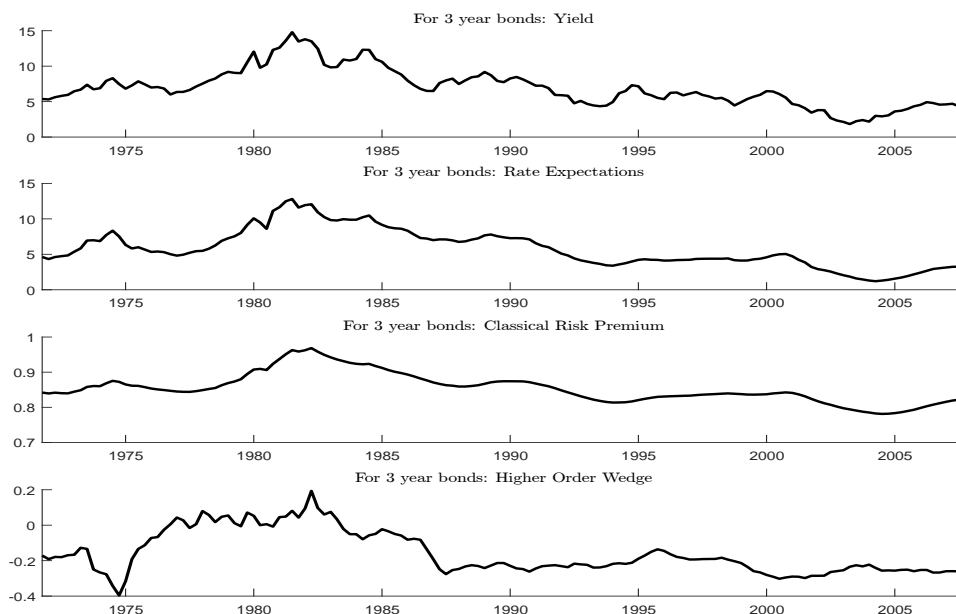
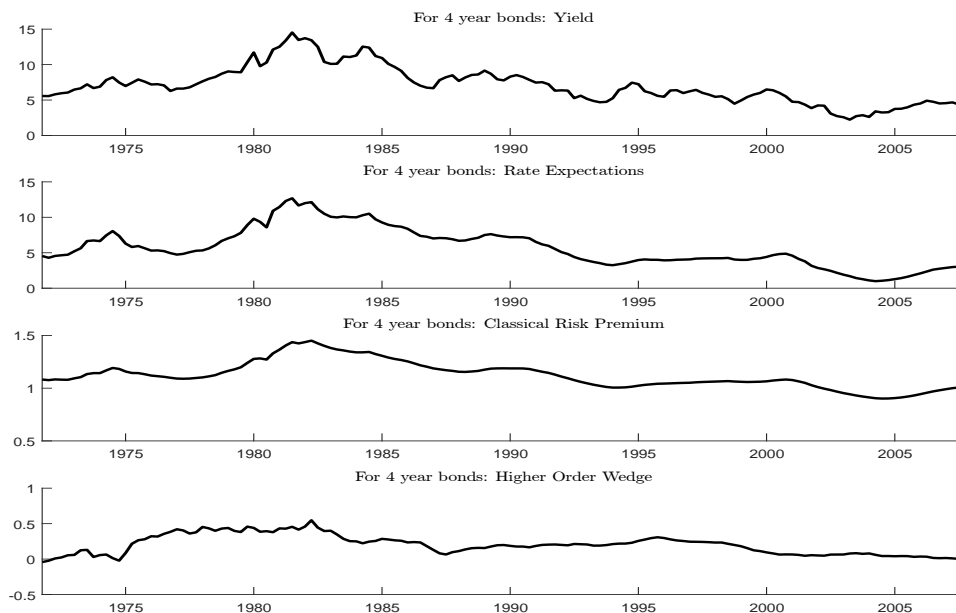## H.3   Yield Decompositions at Posterior Mode
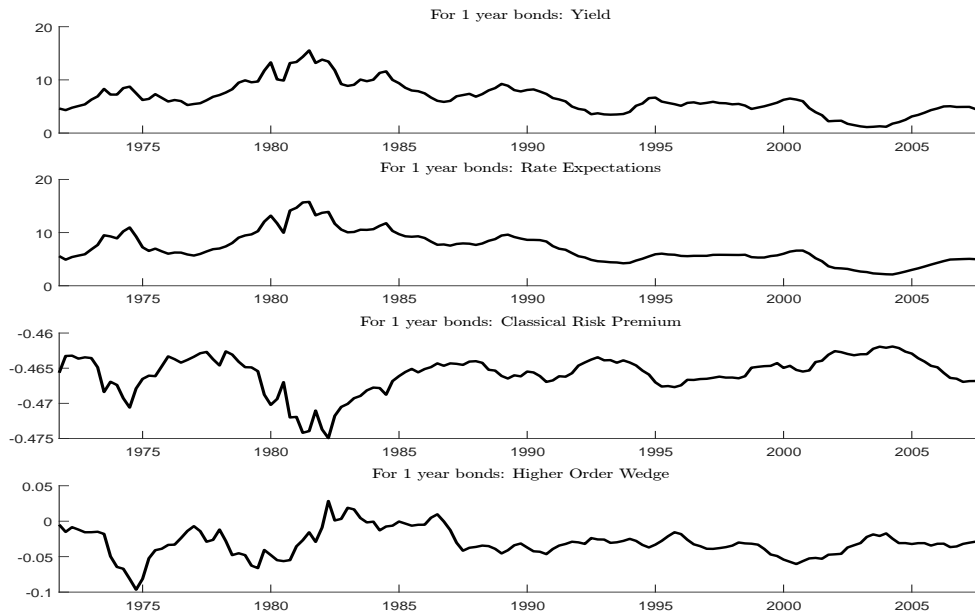
Figure 18: Decomposition of 1 year yields, posterior mode of dispersed information model
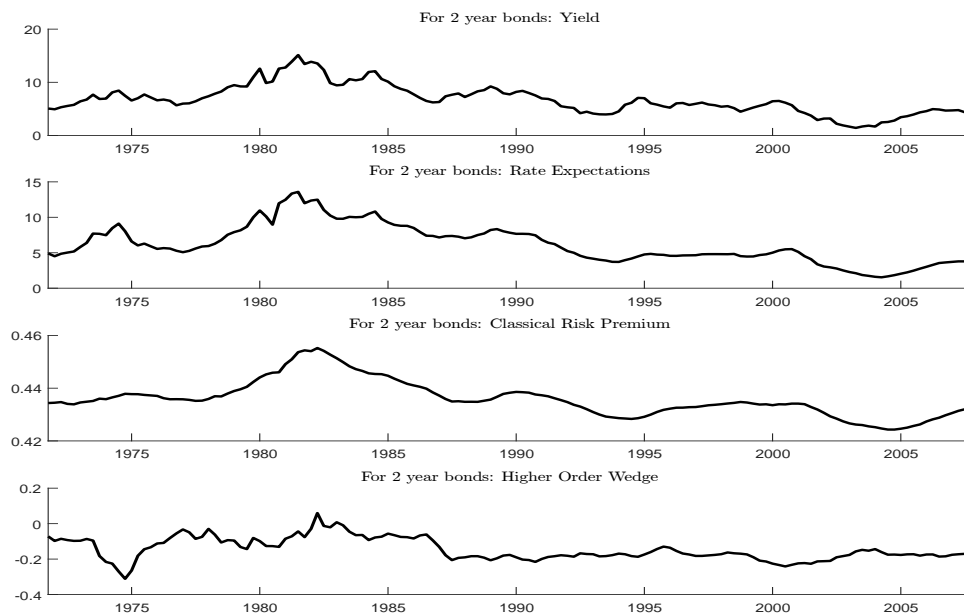
Figure 19: Decomposition of 2 year yields, posterior mode of dispersed information model

Figure 20: Decomposition of 3 year yields, posterior mode of dispersed information model

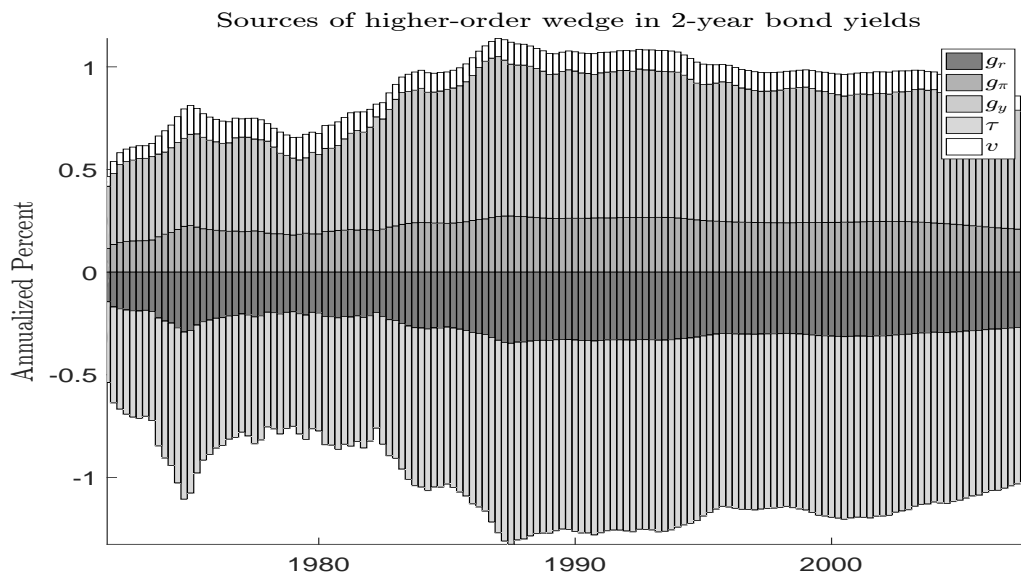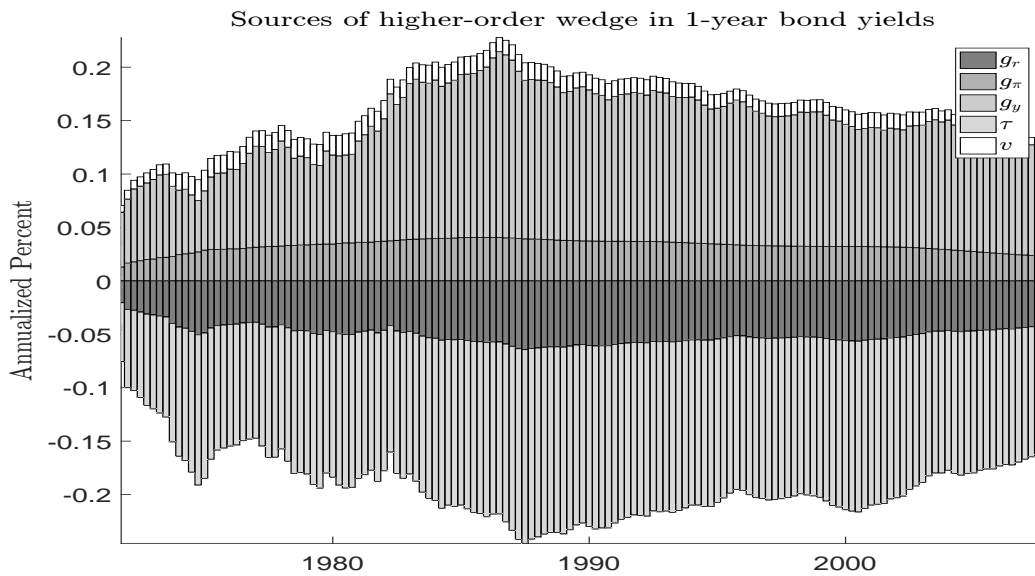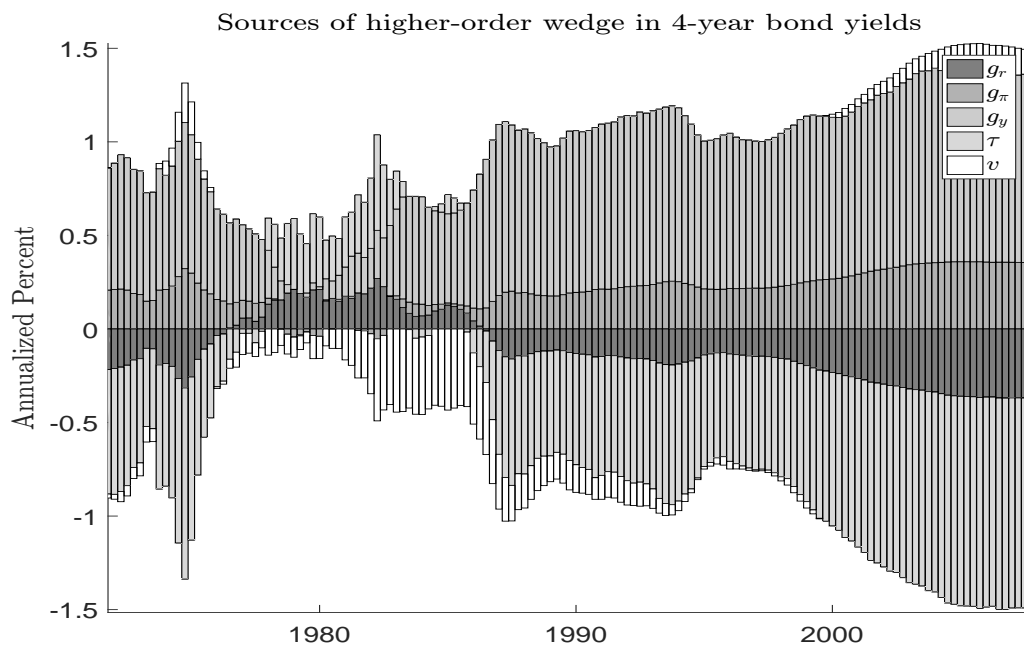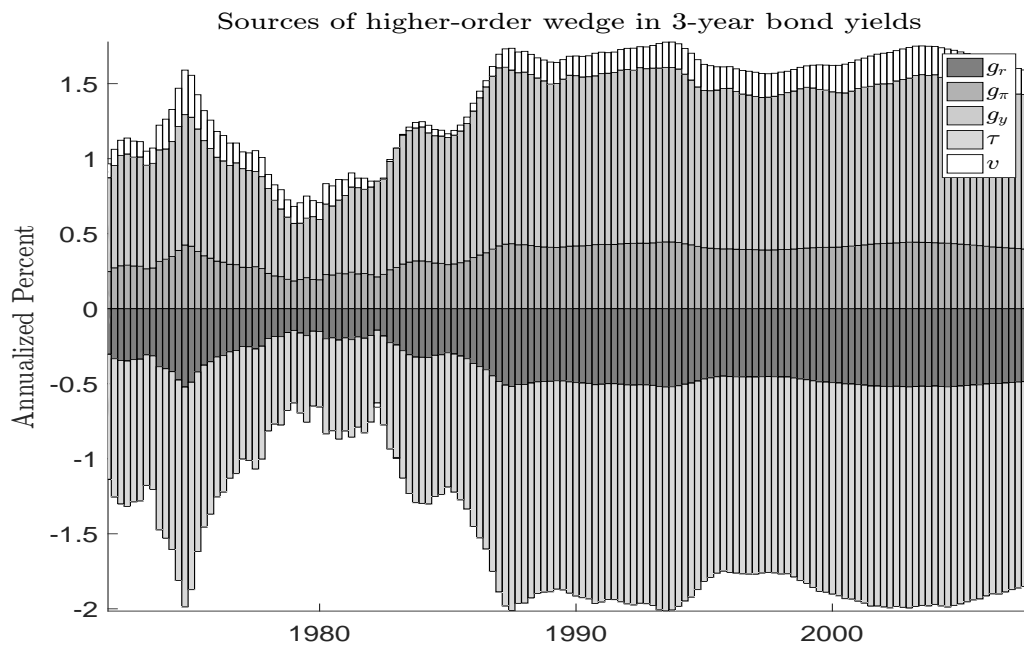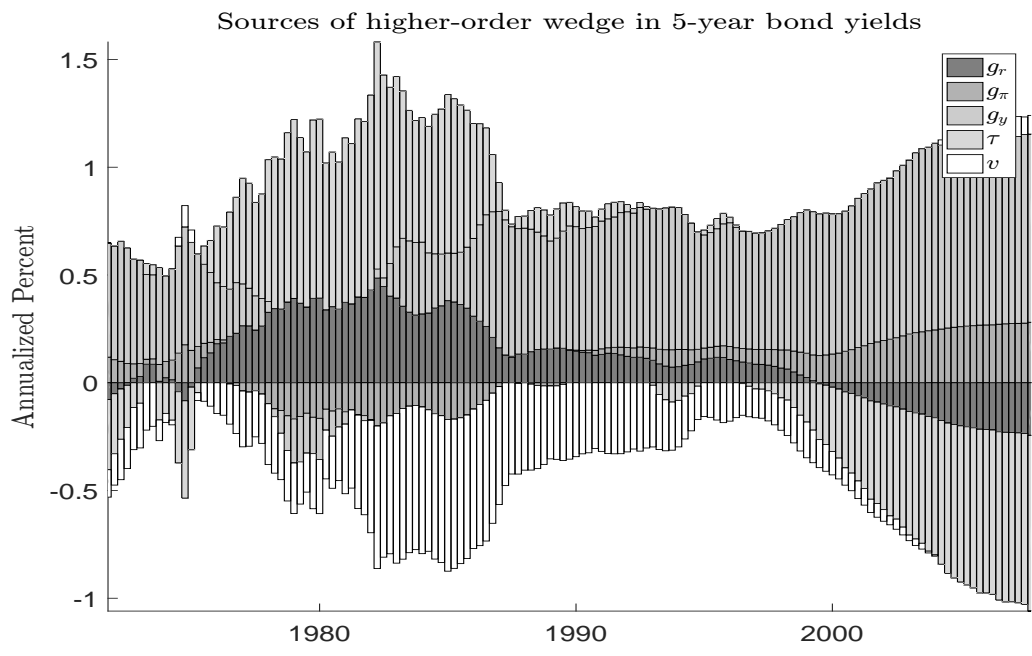Figure 21: Decomposition of 4 year yields,posterior mode of dispersed information model
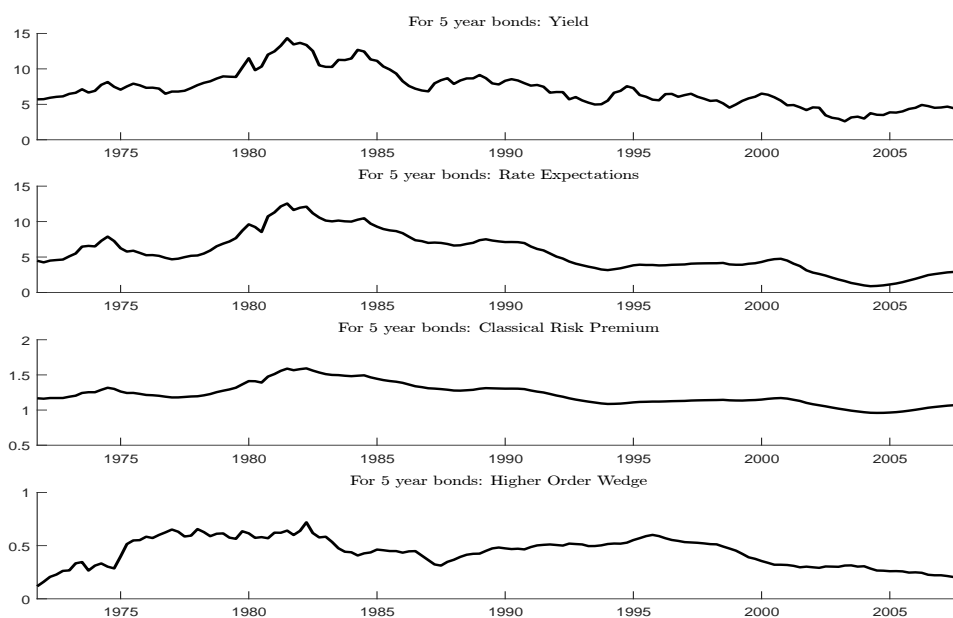
Figure 22: Decomposition of 5 year yields, posterior mode of dispersed information model

H.4 Wedge Decompositions at Posterior Mode

Sources of higher-order wedge in 1-year bond yields

Sources of higher-order wedge in 2-year bond yields

Sources of higher-order wedge in 3-year bond yields

Sources of higher-order wedge in 4-year bond yields

Sources of higher-order wedge in 5-year bond yields

I   Results, full information model

Table 8: Posterior Estimates, Full Information Model

|	Mode	Mean	Median	5th percentile	95th percentile	Std. Dev
ϕ_r	0.5349	0.5373	0.5350	0.5154	0.5727	0.0170
ϕ_π	0.1771	0.1760	0.1702	0.1098	0.2522	0.0435
ϕ_y	0.1178	0.1114	0.1106	0.0908	0.1353	0.0133
ϕ_v	0.0283	0.0221	0.0221	0.0128	0.0314	0.0058
σ_r	0.0020	0.0021	0.0021	0.0019	0.0023	0.0001
ρ_{yr}	-0.9946	-0.9278	-0.9334	-1.1513	-0.6789	0.1454
$\rho_{y\pi}$	-0.3899	-0.4031	-0.4077	-0.5526	-0.2503	0.0943
ρ_{yy}	0.9525	0.9277	0.9266	0.8835	0.9730	0.0272
ρ_{yv}	-0.0013	-0.0149	-0.0131	-0.0368	-0.0011	0.0109
$\sigma_{y\pi}$	0.2903	0.3863	0.3868	0.1419	0.6309	0.1509
$\sigma_{y\tau}$	2.6851	2.6387	2.6360	2.2813	2.9877	0.2174
σ_y	0.0066	0.0068	0.0068	0.0061	0.0075	0.0004
σ_τ	0.0012	0.0012	0.0012	0.0010	0.0013	0.0001
$\rho_{\pi r}$	0.8326	0.8344	0.8280	0.7050	1.0022	0.0831
$\rho_{\pi\pi}$	0.4024	0.4059	0.4057	0.3579	0.4558	0.0299
$\rho_{\pi y}$	-0.1750	-0.1787	-0.1774	-0.2191	-0.1440	0.0235
$\rho_{\pi v}$	-0.0842	-0.0832	-0.0822	-0.1005	-0.0706	0.0088
$\sigma_{\pi\tau}$	-0.1585	-0.1803	-0.1836	-0.3356	-0.0191	0.0956
σ_π	0.0040	0.0040	0.0040	0.0036	0.0044	0.0003
ρ_{vv}	0.8610	0.8550	0.8533	0.8321	0.8931	0.0177
σ_{vr}	9.8381	9.7069	9.7540	9.2463	9.9673	0.2172
$\sigma_{v\pi}$	2.1456	1.9804	2.0660	1.2514	2.5665	0.4220
σ_{vy}	-2.1201	-2.1669	-2.1473	-2.6129	-1.7968	0.2609
$\sigma_{v\tau}$	0.8711	0.8384	0.7641	0.1380	1.6089	0.4336
λ_r	1.2591	1.1900	1.2892	-0.3004	2.2988	0.7652
λ_π	-4.3221	-4.4279	-4.7589	-7.6234	0.1405	2.3402
λ_y	-0.6214	-0.2257	-0.4712	-2.4933	2.3231	1.5115
λ_τ	-0.1198	-0.1123	-0.1139	-0.2503	0.0229	0.0960
λ_r^x	18.3449	18.5892	18.6880	13.1336	23.8793	3.3348
λ_π^x	-76.2955	-77.1364	-76.8624	-80.6353	-73.8037	2.0341
λ_y^x	-17.4764	-22.7287	-21.9104	-33.4025	-17.1115	4.4454
λ_τ^x	0.1536	0.0564	0.0529	-1.7520	1.8245	1.0744
$\widetilde{\sigma}_4$	0.0012	0.0012	0.0012	0.0011	0.0014	0.0001
$\widetilde{\sigma}_8$	0.0011	0.0011	0.0011	0.0010	0.0012	0.0001
$\widetilde{\sigma}_{12}$	0.0009	0.0010	0.0010	0.0008	0.0011	0.0001
$\widetilde{\sigma}_{16}$	0.0010	0.0009	0.0009	0.0008	0.0011	0.0001
$\widetilde{\sigma}_{20}$	0.0010	0.0010	0.0010	0.0008	0.0011	0.0001
$\widetilde{\sigma}_{40}$	0.0014	0.0014	0.0014	0.0012	0.0016	0.0001
$\widetilde{\sigma}_\pi^1$	0.0032	0.0032	0.0032	0.0031	0.0032	0.0000
$\widetilde{\sigma}_\pi^4$	0.0039	0.0039	0.0039	840.0038	0.0039	0.0000
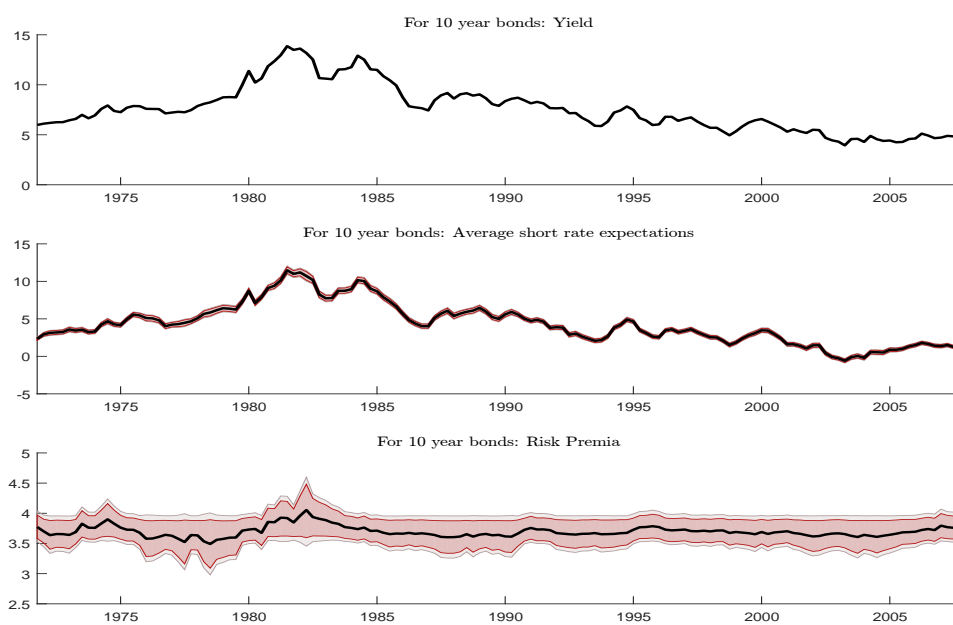
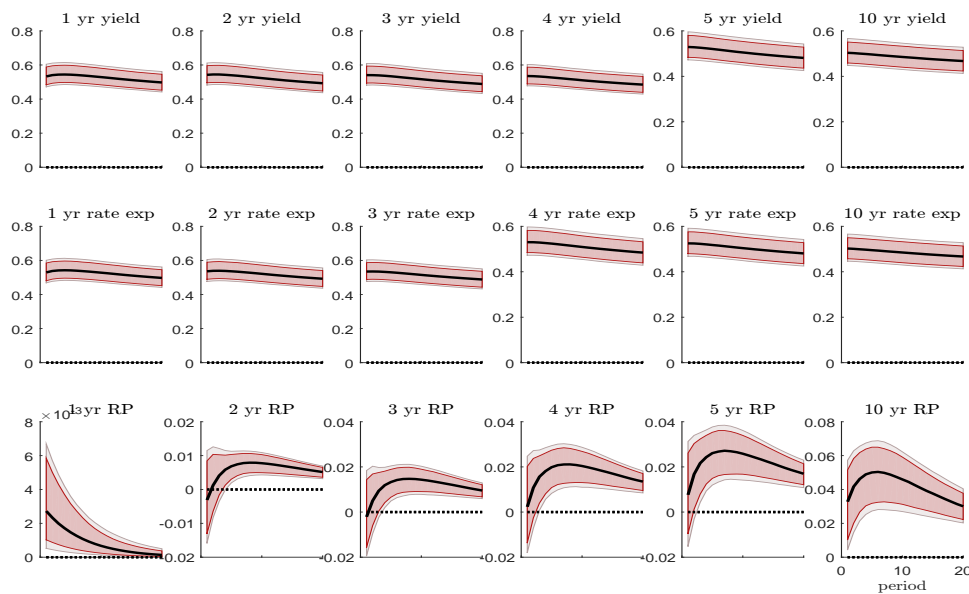Figure 23: Decomposition of 10 year yields, full information
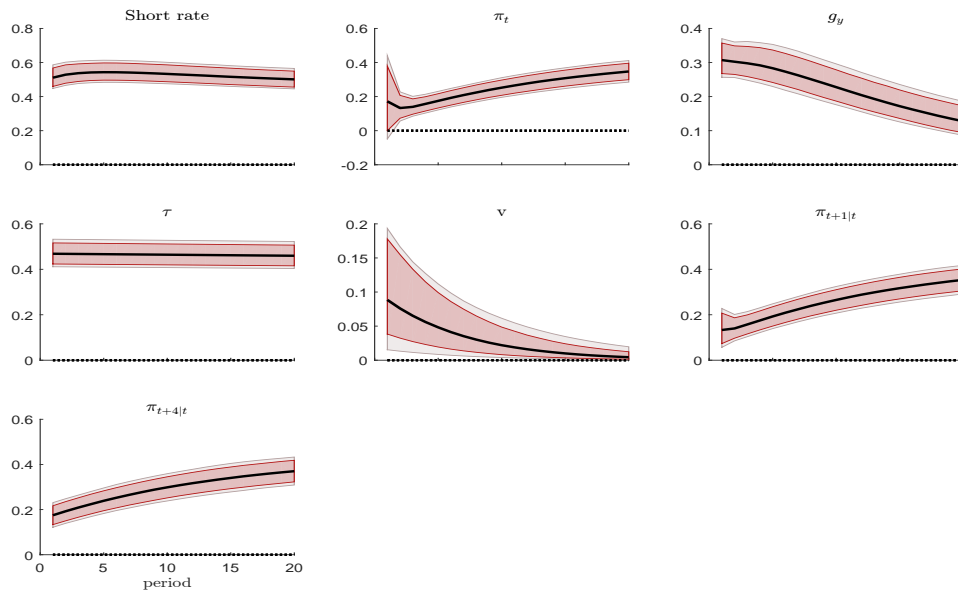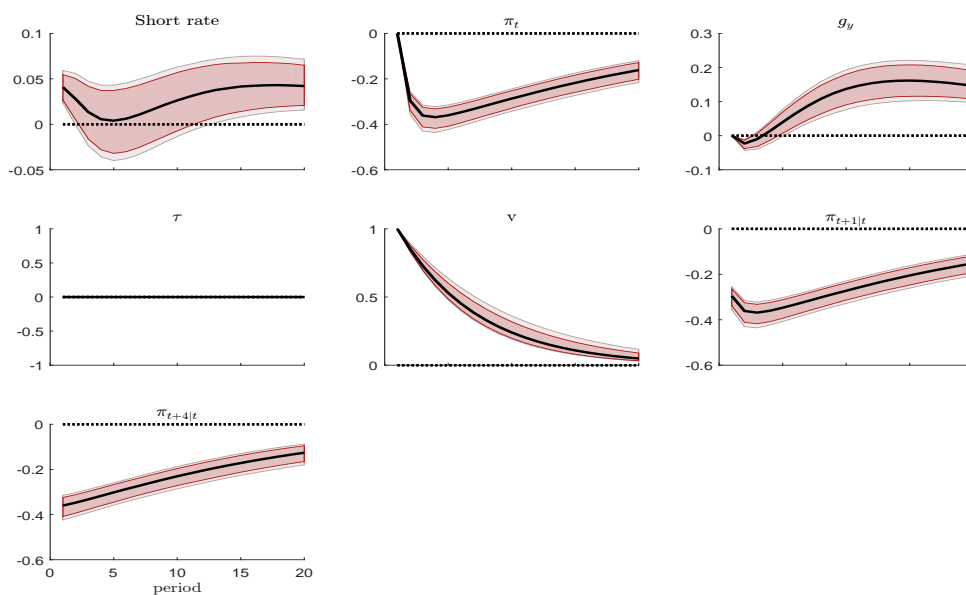
Target shock

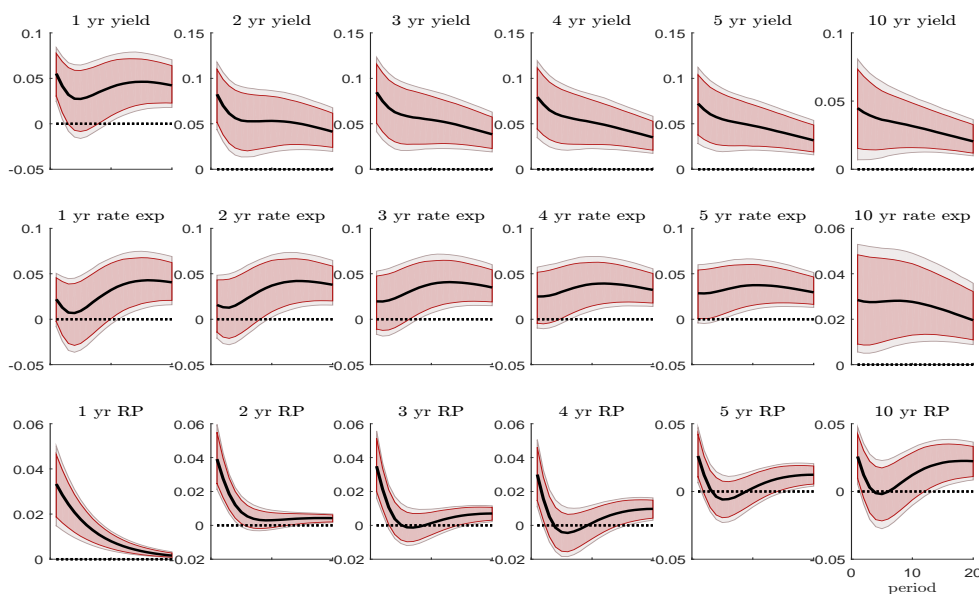(a) Response of non-financial variables to inflation target shock, full information

Target shock

(b) Response of financial variables to inflation target shock, full information

(a) Response of non-financial variables to risk shock, full information

(b) Response of financial variables to risk shock, full information

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